A Low Complexity Algorithm to Simulate the Spatial Covariance Matrix for Clustered MIMO Channel Models

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***Abstract*— The capacity and error rate performance of a multiple-input multiple-output (MIMO) communication system depend strongly on the spatial correlation properties introduced by clustering in the propagation environment. Simulating correlated channels, using the common *correlated Rayleigh fading* model for example, requires numerically complex calculations of the transmit and receive spatial correlation matrices as a function of the cluster size and location. This paper proposes a numerically efficient way of generating these correlation matrices for indoor clustered channel models. This method makes use of an uniform linear array approximation to avoid numerical integrals and derives a closed-form expression for the correlation coefficients, assuming a Laplacian angle distribution. Simulations show that the approximate correlation model exhibits good fit for moderate angle spreads. Complexity calculations show that this approach takes about $1/200$ of the time to compute the spatial correlation matrices compared to existing methods.**

I. Introduction

Multiple-input multiple-output (MIMO) communication technology offers a new spatial degree of freedom that can be leveraged to achieve significant capacity gains as well as improved diversity advantage [1]. While the theoretical properties of MIMO communication systems have been acknowledged for some time, only now is a pragmatic perspective of MIMO communication in realistic propagation channels being developed [2]–[5]. These results show that realistic MIMO channels have significant spatial correlation due to the presence of scatterers in the propagation environment. Unfortunately, spatial correlation generally has an adverse effect on capacity and error rate performance [5], [6]. Simulating realistic correlated channels is thus essential to predict the performance of real MIMO systems.

One common modeling used to simulate MIMO channels is the so called *correlated Rayleigh fading* model [7]. This statistical model determines the spatial covariance matrix assuming certain distributions of the scatterers in the propagation environment. In [8], an exact expression of the spatial correlation coefficients was derived. This solution, however, is expressed as a sum of Bessel functions of the first kind and significant computational complexity is required to complete the calculation.

In this paper, we propose a computationally efficient method for simulating spatial covariance matrices. We focus on the clustered channel model, in which groups of scatterers are modeled as clusters located around the transmit and receive antenna arrays. The clustered channel model has been validated through measurements [9], [10] and has been adopted by various wireless system standards bodies such as the *IEEE 802.11n* Technical Group (TG) [11] and the *3GPP/3GPP2* Technical Specification Group (TSG) [12]. Our proposed method is a practical alternative to the approaches suggested by these standards forums for simulating correlated MIMO channels.

Our specific contribution is to propose a more efficient way of simulating the spatial covariance matrix for the case of Laplacian angular spread distribution. The key insight is that an approximation for uniform linear arrays (ULAs) and moderate angle spreads allows us to derive a closed-form solution for the spatial covariance function, avoiding complicated numerical integrations as in [8]. The main benefit of the proposed method, as we demonstrate, is a dramatic reduction in computational complexity and thus computation time required to calculate the spatial covariance matrices. Because the spatial correlation matrices are a function of the clusters, which are also random, system level simulations will require averaging over many covariance realizations. This computation time improvement of the proposed method makes it particularly suitable in the context of network simulators, where many users and channels need to be simulated. Preliminary results have been shown in [13]. The proposed method to derive the spatial correlation coefficients yields loss in performance compared with the exact method in [8]. We quantify this loss in terms of channel capacity and show that the loss is negligible for spreads lower than 15° and broadside directions.

This paper is organized as follows. In Section II, we provide some background on the clustered channel model and present the system model used for simulations. Section III presents the analytical derivation of the proposed model, outlining the approximation used. In Section IV, we show the performance degradation in terms of channel capacity, due to this approximation. Finally, we present the gain in computational time as

This material is based in part upon work supported by the Texas Advanced Technology Program under Grant Nos. 003658-0614-2001 and 003658-0380-2003, and the National Instruments Foundation.

well as the algorithm complexity analysis. Concluding remarks are given in Section V.

II. Model Description

A. Background on Clustered Channel Models

It is well known that the spatial correlation in MIMO channels depends on the spacing of the receive antennas as well as the spatial characteristics of the propagation environment. Multiple paths are perceived by the receiver as channel taps, each one characterized by a given time delay and angle of arrival (AoA). One common modelling technique is to group these channel taps into clusters, according to the well known Saleh-Valenzuela model [9] for indoor environments. An extension of this model to include angles of arrival has been proposed in [10].

According to this method, a mean AoA (ϕ_c) is associated to each cluster and the AoAs (ϕ_0) of the multi-paths within the same cluster are generated with respect to a certain probability density function (pdf). The pdf of the AoAs is chosen to fit the power azimuth spectrum (PAS) of the channel. The standard deviation of each cluster PAS is a measure of the cluster angular spread (AS). Some channel models (i.e., 802.11n) define multiple resolvable rays within the same channel tap. In this case, each ray is identified by an AoA offset ϕ_i with respect to the mean AoA of the channel tap (ϕ_0). The framework of this model is sketched in Fig. 1.

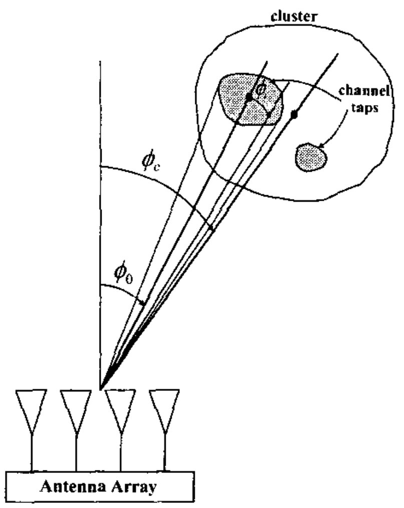

Fig. 1. Geometry of the model representing cluster and channel taps. The angles ϕ_c and ϕ_0 are the mean AoAs of the cluster and channel tap, respectively. The angle ϕ_i is the AoA offset of the channel tap.

The simulation of the spatial correlation matrix relies on the distribution of the AoAs. Various recent measurement campaigns showed the best fit for the PAS to follow a Laplacian probability distribution function [10], [14], [15]. This is the distribution we will refer to throughout this paper.

B. System Model

Consider an M_t transmit and M_r receive antenna wireless system with single user. The MIMO channel matrix is generated on a tap by tap basis, according to the well known *correlated Rayleigh fading* channel model [7], and it can be written as[1]

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{1}$$

where $\mathbf{H}_w$ is M_rxM_t matrix of complex Gaussian coefficients, $\mathbf{R}_r$ and $\mathbf{R}_t$ are the spatial covariance matrices at the receiver and transmitter, respectively, expressing the correlation of the receive/transmit signals across the array elements. We derive a closed-form expression for the coefficients of $\mathbf{R}_r$ and $\mathbf{R}_t$, for a single channel tap characterized by a certain angular spread and angle of arrival. Since the same method is applied to both correlation matrices, we will use the notation $\mathbf{R}$ to refer to both the transmit or receive covariance matrix. Likewise, we will use M, instead of M_r or M_t, to indicate the number of antennas.

To derive the coefficients of the matrix $\mathbf{R}$, we compute the spatial correlation of the signals received at the array sensors. If we neglect the effect of the noise, the signal received at the m-th array element at time t can be written as

$$r_m(t) = s(t)\,\frac{1}{\sqrt{N}}\sum_{i=1}^{N} g_i(t)e^{jDm\sin(\phi_0-\phi_i)} \tag{2}$$

where $m = 0, ..., (M-1)$, M is the number of sensors in the antenna array, N is the number of rays for a given channel tap, $s(t)$ is the complex envelope, and $g_i(t)$ is the complex Rayleigh fading coefficient. We define $D = 2\pi d/\lambda$, with λ being the wavelength, and d the spacing in between the array elements. Moreover, ϕ_i represents the AoA offset with respect to the mean AoA (ϕ_0) of the channel tap, as depicted in Fig. 1.

We assume that each channel tap exhibits a Laplacian PAS. Then, the random variable ϕ, describing the AoA offset with respect to the mean angle ϕ_0, is distributed according to the Laplacian pdf given by

$$P_\phi(\phi) = \begin{cases} \frac{1}{\sqrt{2}\sigma_\phi}\cdot e^{-|\sqrt{2}\phi/\sigma_\phi|} & \text{if } \phi \in (-\pi, \pi]; \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where σ_ϕ is the standard deviation (RMS) of the PAS.

[1] We use $CN(0,1)$ to denote a random variable with real and imaginary parts that are i.i.d. according to $N(0,1/2)$, * to denote conjugation, ^{T} to denote transposition, ^{H} to denote conjugation and transposition, $|\cdot|$ to denote the absolute value, and $\langle\cdot,\cdot\rangle$ to denote the complex vector space inner-product.

III. Proposed Model of the Spatial Correlation Matrix

With the assumptions specified above we compute the cross-correlation between signals at array sensors m and n as

$$\mathbf{R}_{m,n} = E\{r_m r_n^*\} \quad (4)$$

where r_m and r_n is given by (2). We assume the transmitted signal $s(t)$ has power $E_t\{|s(t)|^2\} = 1$ and changes independently of the channel. The complex fading coefficients $g_i(t)$ are assumed to be distributed according to $CN(0,1)$ and are independent over time and from ray to ray. Therefore, $E\{g_i(t)g_k^*(t)\} = \delta_{ik}$, where δ_{ik} is the delta of Kroneker[2]. Moreover, assuming the angles of arrival (ϕ_i) to be independent across different rays, it is possible to express the cross-correlation in (4) as

$$\mathbf{R}_{m,n} = E_\phi\left\{e^{jD(m-n)\sin(\phi_0-\phi)}\right\} \quad (5)$$

where we used the definition given in (2).

Using a first-order Taylor series (assuming $\phi \approx 0$) we can write the exponent in (5) as

$$\sin(\phi_0 - \phi) \approx \sin\phi_0 + \phi\cos\phi_0. \quad (6)$$

Later in this paper, we will discuss the impact of this approximation over the spatial correlation coefficients. Now, substituting (6) into (5) and by the definition of first order moment of a random variable, we get

$$\mathbf{R}_{m,n} \approx e^{jD(m-n)\sin(\phi_0)} \cdot \int_{-\infty}^{\infty} e^{jD(m-n)\cos(\phi_0)\phi}\, P_\phi(\phi)\, d\phi \quad (7)$$

where $P_\phi(\phi)$ is the pdf given in (3).

We observe that the Laplacian PAS decays rapidly to zero within the range $(-\pi,\pi]$, also for high values of RMS AS (i.e., 40^o as in [11]). Therefore, the integration of $P_\phi(\phi)$ truncated over $(-\pi,\pi]$ is equivalent to the one over infinite domain. Then, we can rewrite (7) as

$$[\mathbf{R}(\phi_0,\sigma_\phi)]_{m,n} \approx e^{jD(m-n)\sin\phi_0} \cdot \mathcal{F}_\omega\{P_\phi(\phi)\} \quad (8)$$

where $\mathcal{F}$ denotes the Fourier transform and $\omega = D(m-n)\cos\phi_0$. Using the function in (3), we solve the Fourier transform in (8) as

$$[\mathbf{B}(\phi_0,\sigma_\phi)]_{m,n} = \frac{1}{1+\frac{\sigma_\phi^2}{2}\cdot[D(m-n)\cos\phi_0]^2}. \quad (9)$$

Considering a ULA with array response vector

$$\mathbf{a}(\phi_0) = \left[1\ e^{jD\sin\phi_0}\ \cdots\ e^{jD(M-1)\sin\phi_0}\right]^T, \quad (10)$$

we derive the correlation coefficients across all the array elements. The resulting spatial covariance matrix can be written in closed-form as

$$\mathbf{R}(\phi_0,\sigma_\phi) \approx \left[\mathbf{a}(\phi_0)\cdot\mathbf{a}^H(\phi_0)\right] \odot \mathbf{B}(\phi_0,\sigma_\phi) \quad (11)$$

where $\odot$ denotes the Shur-Hadamard (or elementwise) product. A similar result was derived in [16], where the Gaussian distribution was used for the PAS. Here we computed the matrix $\mathbf{R}(\phi_0,\sigma_\phi)$ for the case of Laplacian pdf, given by (3).

IV. Results & Algorithm Complexity Analysis

Hereafter, we present results showing the performance degradation by using the method given in (11), with the approximation in (6). Then, we discuss the significant gain in computational time of this method versus the one proposed in [11].

A. Performance Results

We simulate the spatial correlation matrix ($\mathbf{R}$) on a cluster-by-cluster basis, as recommended by the 802.11n channel model. We compare three different models to generate the matrix $\mathbf{R}$. The ray-based model (***RBM***) [17], where $\mathbf{R}$ is computed from the steering vectors corresponding to different AoAs. For each tap we generated 5000 rays in order for the distribution of the AoAs to converge to the Laplacian pdf. The second model we analyze is the ***802.11n*** model, based on an approximation with Bessel functions of the first kind as in [11]. The third model is the one using the approximation in (6) which we will refer to as ***Fast-R***, for its high efficiency in computational time. The RBM model is here used as reference for the performance analysis, since it is the most "physical" one and does not use any approximation.

In these simulations we considered a MIMO four transmit and four receive antenna system, with isotropic and equally polarized antennas spaced $\lambda/2$ apart. No coupling effects are considered across the array elements. We measured the performance degradation due to the approximation in (6), by looking at the eigenvectors of the spatial covariance matrices. Then, we compared the dominant eigenvector obtained from the 802.11n model and Fast-R, against the one obtained through the RBM model. We calculated the normalized phase-invariant **(NPI)** distance by defining the distance between two unitary vectors [18], as

$$\mathbf{NPI} = \sqrt{1 - |\langle\mathbf{v}_1,\mathbf{v}_2\rangle|} \quad (12)$$

where $\mathbf{v}_1$ is the dominant eigenvector obtained with the 802.11n or the Fast-R models, whereas $\mathbf{v}_2$ comes from the RBM method. We computed this metric for different values of AS.

The results are depicted in Fig. 2. For both the models the NPI error is due to the assumption of continuous distribution of the rays for each channel tap, as opposed to the RBM model where a finite number of rays are generated. In addition, the

[2]The delta of Kroneker is defined as

$$\delta_{ik} = \begin{cases}1 & \text{if } i = k;\\ 0 & \text{otherwise.}\end{cases}$$

Fast-R method suffers of the approximation in (6). This leads the performance of the Fast-R method to degenerate faster than the 802.11n model, for increasing values of RMS AS. However, for values of AS less than 15^o, the **NPI** for Fast-R is below 2%. We choose this value of AS as reference for the next results.

We computed the cumulative density function (CDF) of the mutual information as function of the signal-to-noise ratio (SNR). We assumed equal power (EP) transmission across the array elements. The mutual information is given by [19]

$$C_{EP} = \sum_{i=1}^{r} \log_2 \left(1 + \frac{\rho}{M_t}\lambda_i\right) \tag{13}$$

where ρ is the average signal-to-noise ratio, and I_{M_r} is the identity matrix with dimensions M_rxM_r. Moreover, we defined $r = \min(M_r, M_t)$ and λ_i to be the eigenvalues of HH^H, where H is the M_rxM_t channel matrix generated as in (1).

In Fig. 3 the CDF of the mutual information is depicted for AS$< 15^o$ and SNR$= 15dB$. It is possible to see that the mean value (or ergodic capacity) matches for all three methods. For the Fast-R, however, the CDF exhibits slightly higher variance. This is due to the fact that the eigenstructure of **R** simulated through Fast-R differs from the one generated through the other two methods, when the AoA approaches the endfire direction with respect to the alignment of the array elements. As consequence, the 10% outage capacity of Fast-R is about $0.5bps/Hz$ lower than the RBM method.

Finally, some remarks on the applicability of Fast-R method to common channel models. In general, the proposed algorithm can be used for any clustered channel model, employing Laplacian distribution. Practical examples are the standards 3GPP/3GPP2 (when statistical method is used) and 802.11n aforementioned. The only constraint required is that the value of AS is lower than 15^o.

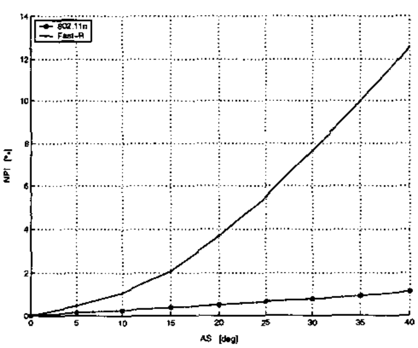

Fig. 2. Normalized phase-invariant (NPI) of the dominant eigenvector of **R** as function of the root-mean squared angular spread (AS), for the "802.11n" and "Fast-R" methods.

B. Computational Complexity Analysis

So far, we have presented the analytical model of Fast-R and its performance degradation compared to some existing methods. Now we show the dramatic improvement in computational

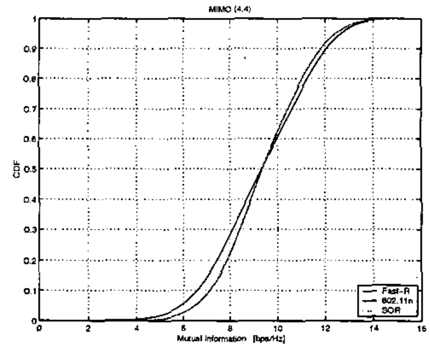

Fig. 3. CDF of the mutual information for tap-AS$< 15^o$ and SNR$= 15dB$. Comparison of different methods to generate **R**.

time obtained with Fast-R as opposed to the RBM and 802.11n approaches. We simulated all three algorithms in Matlab with a $700MHz$ Pentium III personal computer. The 802.11n method is implemented through a series of Bessel functions truncated to the order 100, as recommended by the 802.11n standard model [20]. We generated 6 clusters and a variable number of users. We calculated the computational time for both the methods, and the results are depicted in Fig. 4. It is interesting to see that the Fast-R method is almost 200 times faster than the 802.11n method for any number of simulated users in the system [3]. Then, it is evident the gain of using Fast-R method instead of the 802.11n model to generate covariances for multiple users. This is critical because link and network level simulations must be conduced over long snapshots and many different covariances to measure average system performance.

Finally, we carried out a computational complexity analysis for the Fast-R and the 802.11n algorithms, and compared them. We counted the number of multiplications ($\mathcal{M}$), divisions ($\mathcal{D}$) and additions ($\mathcal{A}$) required to compute the complex spatial correlation coefficients.

For the Fast-R method we referred to the expressions given in (9) and (11). For the 802.11n method we considered the numerical solution of the equations given in [11], for the spatial correlation coefficients. We used the approximation with Bessel functions of the first kind as in [8], truncating the infinite series to the order N_B. We also assumed the use of polynomial approximation and backward recurrence to numerically derive the Bessel functions from the order zero up to N_B, according to well known methods described in [21].

We first derived the computational cost for only one complex coefficient of the spatial correlation matrix, for a given channel tap within a cluster. The results are summarized in Table I.

Since the highest computational complexity is given by the divisions ($\mathcal{D}$), we can approximate the expressions in the ta-

[3]Note that the Fig. 4 shows the computational time needed to calculate only the covariance matrices for the users, without considering the generation of the other channel parameters.

TABLE I

COMPUTATIONAL COST FOR SINGLE CORRELATION COEFFICIENT

Method	Computational Cost ($\mathcal{C}$)
Fast-R	$28\mathcal{M} + 1\mathcal{D} + 14\mathcal{A}$
802.11n	$(11N_B + 8)\mathcal{M} + 2(N_B + 1)\mathcal{D} + (5N_B + 9)\mathcal{A}$

ble by $\mathcal{O}(1)$ for the Fast-R method, and by $\mathcal{O}(2N_B)$ for the 802.11n method. To guarantee a good approximation of the correlation coefficients when using the 802.11n method, we could plausibly truncate the series of Bessel functions to the order $N_B = 100$ as in [20]. For this value, the Fast-R method turns out to be 200 times faster than 802.11n, consistently with the results showed in Fig. 4.

Now, to derive the spatial channel matrix as in (1) we need to compute all the coefficients of the receive and transmit covariance matrices, with dimensions M_rxM_r and M_txM_t, respectively. Moreover, we assume the propagation environment to be characterized by N_c clusters per user, with N_{tap} channel taps each one, and N_{users} in the system. Then, the computational cost for the spatial channel matrix in multiple user scenario is

$$\mathcal{C}_{user} = M_t^2 M_r^2 N_{tap} N_c N_{users}\, \mathcal{C} \qquad (14)$$

where $\mathcal{C}$ is the computational cost given in Table I for both the methods discussed before.

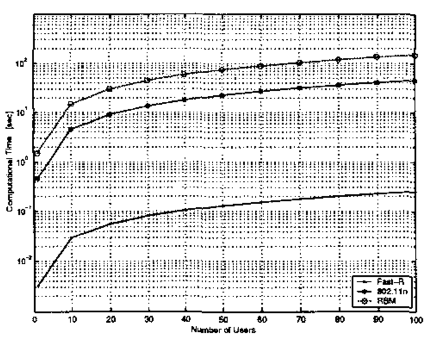

Fig. 4. Computational time as function of the number of users in the system, for "RBM", "802.11n" and "Fast-R" methods.

V. CONCLUSIONS AND FUTURE WORK

We presented a new approach (Fast-R) to simulate the spatial covariance matrix for MIMO systems, for the case of Laplacian distributed PAS and ULA. The Fast-R method is a practical alternative to computing the covariance **R**, according to the 802.11n [11] and 3GPP [12] spatial channel models. The Fast-R method seems to generate covariances that are close to that generated by [11], for angle spreads less than 15^o. The computational reduction is significant.

Further work is needed to investigate any differences between correlation functions in terms of bit error rate. Finally, possible extension to this contribution may be to derive closed-form expressions of the spatial correlation coefficients for other antenna array configurations.

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