Practical Costa Precoding for the Multiple Antenna Broadcast Channel

Manish Airy, Antonio Forenza, Robert W. Heath Jr., Sanjay Shakkottai
Wireless Networking and Communications Group
Department of Electrical and Computer Engineering
The University of Texas at Austin, Austin, TX 78712.
{airy, forenza, rheath, shakkott}@ece.utexas.edu

***Abstract*— For a multiple antenna broadcast channel, the sum-rate capacity achieving transmit strategy requires the centralized transmitter to simultaneously communicate with multiple receivers. The objective of this paper is to design an implementable sum-rate capacity achieving transmit strategy that uses a combination of beamforming and coding for known interference. For a Gaussian broadcast channel with two transmit antennas and two receivers with one antenna each, results indicate that with QAM constellations there is significant gain in sum-rate capacity over an approach that uses only beamforming.[1]**

I. Introduction

Consider a Gaussian BC with K noncooperating receivers, each equipped with M_r antennas. The transmitter has M_t antennas. Perfect channel state information is assumed available at the transmitting base station. The total transmission power (sum power) is bounded. Such a channel belongs to the class of non-degraded BC for which the general capacity region is not yet known. While the general capacity region of MIMO BCs is still an open problem in the research community, recent information theoretic results have demonstrated achievability of the sum-rate capacity of Gaussian MIMO BC.

A partial theoretical solution was explained in [1] for the special case of two users, two transmit antennas and one receive antenna per user. The result shows that throughput-wise optimal (i.e. sum-rate capacity achieving) transmission is possible by using a combination of linear spatial pre-filtering (beamforming) and coding for non-causally known interference at the transmitter (Costa Precoding or "dirty paper" coding, DPC) [2], [3]. Beamforming decomposes the BC into a series of sub-channels, where the interference to subsequent sub-channels is known. Then, DPC is used to mitigate the effect of this known interference.

A practical implementation of DPC first appeared as Tomlinson- Harashima (TH) precoding [4], [5] over the inter-symbol interference (ISI) channel, where a single stream exhibits self-interference across time. A modulo operation was used to reduce signal excursions in order to reduce power enhancement produced by DPC. The one-dimensional modulo technique was generalized to multi-dimensional vector quantization (VQ) in [6], and combined with trellis precoding for the MIMO BC. VQ is also applied in [7] where nested lattices are used both for the inter-symbol interference channel and the general multiuser BC. In [7], the authors distinguish two cases where the channel information and interference is known causally ("dirty tape" coding) or non-causally ("dirty paper" coding). Capacity achieving strategies are only known for the case where channel state information and interference are known non-causally at the central transmitter. The capacity for the causal case, as was Shannon's original formulation, remains an open problem, though the same transmit strategy as the non-causal case, is shown to be asymptotically optimal (at high SNR) even for the causal case, [7].

While DPC methods have existed since the 1970s [4], [5] it was only recently discovered that the sum-rate capacity achieving transmit strategy for the MIMO BC is a combination of controlled beamforming that allows some interference to remain between users followed by an application of DPC to mitigate the remaining interference [1], [8].

Pre-coding based sum-rate capacity studies for the BC have largely been information-theoretic in nature. We attempt to use some of these mostly theoretical ideas and implement a practical design for vector broadcast channels and study the sum-rate performance using QAM constellations.

1) We explicitly implement a "dirty tape" coding strategy. Our strategy uses coding for causally known interference at the transmitter in the context of practical QAM constellations for signaling. At each receiver, we implement a slicer that allows detection of the precoded symbols. The slicer algorithm may be considered an implementation of superposition coding [11]. Through simulations we explicitly evaluate symbol-error rate performance for different SNRs and QAM constellations. Related work in [9] combines analysis of pre-coding and beamforming. However, [9] does not consider sum-rate optimization as the design goal, instead, the constraint is to fulfill individual target SINRs.
2) We compare pure beamforming (or linear beamforming) with our approach that couples beamforming with transmit precoding (or non-linear beamforming). Transmit beamforming is a well-studied subject [13], [14], [15].

[1]This material is based in part upon work supported by the Texas Advanced Technology Program under Grant No. 003658-0380-2003, NSF Grants ACI-0305644 and CNS-0325788, and the Samsung Advanced Institute of Technology.

Sum-rate optimal linear beamforming cannot be formulated as a convex optimization problem and hence there is no obvious method to find the corresponding sum-rate achieving linear beamforming vectors [15]. Thus, the only linear beamforming technique we compare against is the zero-forcing beamforming (ZF-BF) [13].

A. Signal Model

We consider a base station (or central transmitter) with $M_t = 2$ transmit antennas and 2 receivers with $M_r = 1$ antennas each. Frequency flat Rayleigh fading channels are assumed. The total transmit power constraint is P_t. Figure 1 illustrates the framework we analyze. We consider square M-QAM constellations; extensions to non-square QAM constellations is straightforward.

II. Joint Beamforming and Transmit Precoding Algorithm

Next, we propose a practical implementation of the dirty-tape precoding method. We derive the sum-rate optimizing beamforming vector and construct the "dirty-tape" encoded transmit waveform. At the receiver, we propose a slicing algorithm that can decode the precoded waveform.

Let s_1 and s_2 denote the transmit symbol, drawn from possibly distinct constellations, for user 1 and user 2 respectively. Let $\mathbf{h}_1$ and $\mathbf{h}_2$ be the channel vectors for user 1 and user 2 respectively, denoted $\mathbf{h}_1$, $\mathbf{h}_2 \in \mathbb{C}^{2\times 1}$ respectively. Denote by $\mathbf{w}_1$ and $\mathbf{w}_2$ the unitary transmit beamforming vectors for user 1 and user 2 respectively, $\mathbf{w}_1$, $\mathbf{w}_2 \in \mathbb{C}^{2\times 1}$. Without any precoding the transmitted signal is given by

$$\mathbf{x} = \sqrt{E_1}\,\mathbf{w}_1 s_1 + \sqrt{E_2}\,\mathbf{w}_2 s_2, \quad (1)$$

where E_1 and E_2 are the allocated powers to user 1 and user 2, respectively, with $P_t = E_1 + E_2$. In the presence of Gaussian noise, the received signal at the two receivers is given by

$$y_1 = \mathbf{h}_1{}^T(\sqrt{E_1}\,\mathbf{w}_1 s_1 + \sqrt{E_2}\,\mathbf{w}_2 s_2) + n_1, \quad (2)$$

$$y_2 = \mathbf{h}_2{}^T(\sqrt{E_1}\,\mathbf{w}_1 s_1 + \sqrt{E_2}\,\mathbf{w}_2 s_2) + n_2. \quad (3)$$

A. Dirty Tape Precoding

Observe from (2) and (3) that if the dot-products, $\langle \mathbf{h}_1{}^*, \mathbf{w}_2\rangle$ and $\langle \mathbf{h}_2{}^*, \mathbf{w}_1\rangle$, are non-zero then both user 1 and user 2 experience interference from the other users transmission. Dirty tape precoding transforms the transmit signal in (1) such that one of the users sees no interference from the other. In particular let us choose user 2 to be interference free from user 1 transmissions, then dirty tape precoding takes the following form in constructing the transmit waveform

$$\tilde{\mathbf{x}} = \sqrt{E_1}\,\mathbf{w}_1 s_1 + \mathbf{w}_2\left[\sqrt{E_2}\,s_2 - \sqrt{E_1}\,\frac{\langle \mathbf{h}_2{}^*, \mathbf{w}_1\rangle}{\langle \mathbf{h}_2{}^*, \mathbf{w}_2\rangle} s_1\right]. \quad (4)$$

With dirty tape precoding, the received signals at the two receivers are given by

$$\tilde{y}_1 = \sqrt{E_1}\,\mathbf{h}_1{}^T\left[\mathbf{w}_1 - \frac{\langle \mathbf{h}_2{}^*, \mathbf{w}_1\rangle}{\langle \mathbf{h}_2{}^*, \mathbf{w}_2\rangle}\mathbf{w}_2\right] s_1 + \sqrt{E_2}\,\mathbf{h}_1{}^T\mathbf{w}_2 s_2 + n_1, \quad (5)$$

$$\tilde{y}_2 = \sqrt{E_2}\,\mathbf{h}_2{}^T\mathbf{w}_2 s_2 + n_2. \quad (6)$$

Note that the received signal at user 2, $\tilde{y}_2$, is independent of user 1 data symbols. Inspecting (4) closely, note that an appropriate and scaled projection of user 1 data symbols was presubtracted from user 2's transmit signal, resulting in user 2's received signal being orthogonal to user 1.

B. Sum-Rate Capacity Optimizing Beamforming Vectors

In section II-A we discussed how known interference can be presubtracted at the transmitter for user 2. It was assumed that the transmit beamforming vectors and power allocations for user 1 and user 2 were known. Now we show how the sum-rate capacity optimizing beamforming vectors and power allocations are obtained.

The sum-rate capacity for a broadcast channel with M_t transmit antennas and K receivers each with M_r antennas is achieved by finding a set of optimal covariance matrices, $\mathbf{R}_k \in \mathbb{C}^{M_r\times M_r}$ where $k = 1, 2, ..., K$, for the dual MAC channel that maximize

$$C_{SumRate} = \max_{\mathbf{R}_k\geq 0, \sum_k Tr(\mathbf{R}_k)\leq P_T} \log_2\left|\mathbf{I} + \sum_k \mathbf{H}_k^\dagger \mathbf{R}_k \mathbf{H}_k\right| \quad (7)$$

where k indexes the users, $\mathbf{H}_k \in \mathbb{C}^{M_r\times M_t}$ is the downlink channel for user k, $\mathbf{R}_k \geq 0$ indicates that the dual MAC transmit covariance matrices are positive semi-definite, [8]. Once the sum-rate optimal dual MAC covariances, $\mathbf{R}_k$, are obtained the optimum broadcast transmit covariance matrices, $\mathbf{Q}_k$, are found using the duality transformations in [8].

For the $M_t = 2$, $M_r = 1$ case we consider, $\mathbf{Q}_k \in \mathbb{C}^{2\times 2}$ and $\mathbf{Q}_k^\dagger = \mathbf{Q}_k$. The power allocated to user k is the trace of $\mathbf{Q}_k$, $Tr(\mathbf{Q}_k)$, and $\mathbf{Q}_k$ is rank 1. Denote the eigenvalue decomposition of $\mathbf{Q}_k$ by $\mathbf{Q}_k\mathbf{V}_k = \mathbf{V}_k\mathbf{D}_k$, where $\mathbf{D}_k \in \mathbb{R}^{2\times 2}$ is a diagonal matrix of eigenvalues and $\mathbf{V}_k \in \mathbb{C}^{2\times 2}$ is a matrix whose columns contain the eigenvectors of $\mathbf{D}_k$. The optimum transmit beamforming vectors and power allocations for user are given by

$$\mathbf{w}_1 = \mathbf{v}_1^{(1)}, \quad \sqrt{E_1} = d_1^{(1)} \quad (8)$$

$$\mathbf{w}_2 = \mathbf{v}_2^{(1)}, \quad \sqrt{E_2} = d_2^{(1)} \quad (9)$$

where $d_k^{(1)}$ and $\mathbf{v}_k^{(1)}$ are the eigenvalue and corresponding eigenvector for user k (since $\mathbf{Q}_k$ is rank 1, there is only one non-zero eigenvalue for each user k).

Figure 2 illustrates how the optimum beamforming vector obtained by the sum-rate capacity maximization compares with other techniques like: i) transmit max ratio combining (MRC) and ii) zero forcing (ZF) with interference nulling.

- Transmit MRC: For each user, the BTS chooses the individual user SNR maximizing solution. The curve with crosses forms a beam radiation pattern with a maximum at +60. The curve with stars forms a beam that peaks at −60. Note however, that the interference to each user from the other is uncontrolled and high.
- Linear ZF-Beamforming: Here the weight vector is chosen such that a null is placed towards the direction of the interferer, [13]. The curves with points and circles plot the corresponding optimal solution. Note that both the curves

form beam patterns that peak not at the user locations but in a direction that balances interference generated to the other user.

- Sum-Rate Optimizing Beamforming: Here the optimum beamforming vectors from (8) and (9) were used. The resulting beam radiation patterns formed are indicated in black. Interestingly, the beam pattern for user 2 is the transmit MRC solution for user 2 and the beam pattern for user 1 is the ZF-Beamforming solution. User 2 experiences no interference from user 1. However, user 1 sees interference from user 2's beam radiation pattern.

C. Power Enhancement

The subtraction in (4) may result in a power enhancement at the transmitter. To keep the range of signal excursions limited and hence restrict the power enhancement there are two options:

1) Non-linear Precoding: Suppose that the transmitter uses a M-QAM constellation with adjacent symbols 2ρ units apart. For user 2 since the central transmitter has complete information of the interference from user 1, as described in Section II-B, this known interference for user 2 may be pre-subtracted prior to transmission. For user 2 the effect of this interference subtraction is as if the original M-QAM constellation was "expanded" on the complex plane. The received vector is a noisy version of the modulo-equivalent transmitted signal. To recover the original data symbol, the receiver either does another modulo operation prior to detection or uses a slicer based on the expanded constellation.

The above description is that of the classical Tomlinson-Harashima (TH) precoder for ISI channels, [4], [10]. Figure 3 illustrates how TH precoding works. To restrict the power enhancement due to pre-subtraction, the transmitted symbol vector in (4) may be modified

$$\tilde{\mathbf{x}} = \sqrt{E_1}\,\mathbf{w}_1 s_1 + \mathbf{w}_2\left[\left(\sqrt{E_2}\,s_2 - \sqrt{E_1}\,\frac{\langle \mathbf{h_2}^*, \mathbf{w}_1\rangle}{\langle \mathbf{h_2}^*, \mathbf{w}_2\rangle} s_1\right) \oplus \mathcal{M}\right], \quad (10)$$

where $\oplus$ denotes the modulo operation that brings the pre-subtracted signal back to the fundamental region of user 2s constellation denoted by $\mathcal{M} = \{(-M\rho, M\rho] \times (-M\rho, M\rho]\}$, as illustrated in Figure 3. The TH precoder produces an effective transmit symbol that can take any value in the fundamental region for user 2, $\mathcal{M} = \{(-M\rho, M\rho] \times (-M\rho, M\rho]\}$, [10], and causes a transmit power penalty, called precoding power loss.

2) Linear Precoding: As an alternative to TH precoding, note that the transmitted symbol vector in (4) may be rearranged as

$$\tilde{\mathbf{x}} \quad = \quad \frac{\sqrt{E_1}}{\Gamma(\gamma)}\left[\mathbf{w}_1 - \frac{\langle \mathbf{h_2}^*, \mathbf{w}_1\rangle}{\langle \mathbf{h_2}^*, \mathbf{w}_2\rangle}\mathbf{w}_2\right] s_1 + \sqrt{E_2}\,\mathbf{w}_2\, s_2, \quad (11)$$

where $\Gamma(\gamma) = ||\mathbf{w}_1 - \gamma\mathbf{w}_2||$ and $\gamma = \frac{\langle \mathbf{h_2}^*, \mathbf{w}_1\rangle}{\langle \mathbf{h_2}^*, \mathbf{w}_2\rangle}$, with $\gamma \in \mathbb{C}$. Note that $\Gamma(\gamma)$ is chosen to satisfy the sum power constraint $P_t = \mathbb{E}\left[\tilde{x}^\dagger \tilde{x}\right] = E_1 + E_2$. Therefore, no precoding power loss is produced. This is the linear power scaling precoding.

D. Detection

Both non-linear TH-precoding and the linear power scaling precoding result in the received signal at user 2 being orthogonal to user 1 transmission. At the receiver user 2 first compensates for the gain and phase in the effective channel $\sqrt{E_2}\mathbf{h}_2^T\mathbf{w}_2$. Then, if TH-precoding is used, user 2 passes the received signal through the same modulo-M operator as the transmitter and then does a symbol by symbol detection. If instead linear power scaling precoding is used, direct symbol by symbol detection using a traditional M-QAM slicer is sufficient.

User 1 needs to implement a more complex receiver. Unfortunately, the modulo operation in (10) introduces a non-linear distortion in the transmit signal which is not easy to compensate at the receiver. The linear power scaling precoding in (11) is somewhat easier to decode. The decoding approach we propose next assumes linear power scaling precoding and may be viewed as a practical implementation of superposition coding, [11]. The proposed algorithm exploits the structure of the QAM constellations during decoding. With linear power scaling precoding, the received signal for user 1 is given by

$$\begin{aligned}\tilde{y}_1 &= \mathbf{h}_1{}^T\tilde{\mathbf{x}} + n_1 \\ &= \frac{\sqrt{E_1}}{\Gamma(\gamma)}\,\mathbf{h}_1{}^T\left[\mathbf{w}_1 - \frac{\langle \mathbf{h_2}^*, \mathbf{w}_1\rangle}{\langle \mathbf{h_2}^*, \mathbf{w}_2\rangle}\mathbf{w}_2\right] s_1 + \sqrt{E_2}\,\mathbf{h}_1{}^T\mathbf{w}_2 s_2 + n_1.\end{aligned} \quad (12)$$

To understand how user 1 may decode the received signal, note that (12) may be written as

$$\tilde{y}_1 = \alpha s_1 + \beta s_2 + n_1, \quad (13)$$

where $\alpha \in \mathbb{C}$, $\beta \in \mathbb{C}$. Denote $\rho_\alpha = |\alpha|$, $\rho_\beta = |\beta|$ and $\varphi_\alpha = \angle\alpha$, $\varphi_\beta = \angle\beta$. Figure 4 explains the slicing operation in detail. The decision regions that user 1 needs are expressed in terms of $\rho_\alpha, \rho_\beta, \varphi_\alpha$, and φ_β. The transmitter has to share knowledge of channel dependent parameters $\rho_\alpha, \rho_\beta, \varphi_\alpha$, and φ_β with user 1. It is important to note that the detection for user 1 is only partially coupled with user 2's transmit signal and that user 1 does not require knowledge of *every* transmit symbol for user 2, as in successive interference cancellation. If the channel coherence time is much larger than the symbol period, it is reasonable to assume that the transmitter aids user 1 detection by coordinating knowledge of $\rho_\alpha, \rho_\beta, \varphi_\alpha$, and φ_β with user 1. Power scaling precoding preserves the shape of the user constellations permitting a structured decoding algorithm. Non-linear TH precoding does not preserve the shape of the user constellations and makes decoding harder.

Figure 4 illustrates the received signal at user 1 over many symbol periods for a fixed channel realization. In general, the received signal points for user 1 may not be easily "separable". Figure 4 illustrates the resulting overlap of received signal points causing decoding ambiguity. In such scenarios, depending upon the transmit symbol for user 2, the constellation for user 1 is adapted or "re-labelled" such that overlapping points in the received signal at user 1 map to the same transmit symbol, thus reducing symbol error probability. Additional coordination between the central transmitter and the receiver is required. This is called adaptive constellation switching, [10].

III. Numerical Results

In this section we present numerical results to illustrate the performance of combined precoding and beamforming.

Figure 5 plots the capacity versus SNR. The first set of plots in the figure plot the *ideal* bandwidth-normalized Shannon capacities for the following scenarios[2],

- SU-MRC Ideal: Here, the capacity is calculated assuming a that even though there are two users in the system, individual user transmissions do not interfere with each other. The available power P_T is equally divided among the two users.
- MIMO 2x2
  - WF (waterfilling): A system with $M_t = 2$, and $K = 2$ with $M_r = 1$ receive antenna each may be considered a $M_t = 2,\ M_r = 2$ MIMO system with *cooperating* receivers, [6]. The capacity of a MIMO link is obtained by waterfilling. This case represents a *strict* upper bound on the sum-rate capacity for two users.
  - EP (equal power): Instead of waterfilling, if no channel knowledge is available at the transmitter, then the ergodic capacity maximizing strategy is equal power per eigenmode.
- SRO Ideal: This case represents the ideal sum-rate capacity of a broadcast channel with $M_t = 2$ and two-receivers each with $M_r = 1$. Notice that SRO Ideal performs better than MIMO 2x2 EP but worse than MIMO 2x2 WF, and at low SNRs, SRO Ideal out-performs SU-MRC Ideal.

Shannon capacity gives the maximum error-free *capacity* for a given SNR. However, Shannon capacity can only be achieved under ideal conditions with infinite data block length. Practical system designs will experience occasional channel introduced errors in the data transmission. In such cases, we may adopt the following measure of performance which we call "throughput" and measure in the same units as bandwidth normalized channel capacity (bps/Hz),

$$C_{bps/Hz}(SNR) = (1 - \mathbb{P}_{SER}(SNR)) \log_2 M,$$

where M is the constellation size, $\mathbb{P}_{SER}$ is the symbol error probability for a given SNR.

The second set of plots in Figure 5 plot "throughput" vs. SNR. In addition to SU-MRC, we consider the following scenarios,

- ZF-BF: This method involves joint (for both users) SINR maximization, [13]. SINR maximization is an effective method of spatial nulling for space division multiple access, also described in Figure 2.
- DTC-SRO: This method uses sum-rate optimal beamforming vectors combined with dirty tape precoding.

Observe that DTC-SRO outperforms ZF-BF for both 4-QAM and 16-QAM. Again, for comparison, we plot the SU-MRC for comparison. At low SNRs DTC-SRO yields upto a factor of 1.5 in average "throughput" over ZF-BF. Alternately, with DTC-SRO, a given "throughput" is achieved at upto 5 dB lower transmit power than ZF-BF.

[2]SU-MRC: single user max ratio combining, SRO: sum rate optimizing

It is also of some interest to compare "throughput" with the ideal Shannon capacity. However, to be able to compare the simulated "throughput"curves with the ideal Shannon capacity, we must account for i) excess bandwidth and ii) shaping loss. In typical wireless systems that do no signal shaping, we can expect a 1.53 dB shaping loss and ≈ 1 dB of excess bandwidth due to pulse shaping, [10]. Thus, in the figure the simulated curves with QAM constellations were penalized by 2.53 dB to allow a comparison with the ideal capacity curves.

IV. Concluding Remarks

A practical Costa precoding method was presented in this paper. We started with obtaining the sum rate optimizing beamforming vectors from the sum-rate optimizing covariance matrices. Then, in a $M_t = 2,\ M_r = 1$, two-user scenario, we designed a practical "dirty-tape" precoding strategy. Without loss of generality we chose user 2 to be interference free from user 1 transmissions. Conditioned on channel knowledge, this precoding strategy pre-subtracts an appropriate projection of user 1's symbols from user 2's transmit symbol such that user 2s receiver sees no interference from user 1 transmissions. This pre-subtraction may result in an energy enhancement and hence TH precoding or power scaling precoding is required at the transmitter to restrict signal excursions and hence satisfy the given transmit power constraint. For either case, the detection algorithm for user 2 is a conventional QAM slicer, or alternately a modulo-extended slicer. User 1 requires a more complicated receiver design and in particular requires that the transmitter coordinate slicing parameters with user 1. Such a slicer design was presented, and an optimization using adaptive QAM constellations was suggested.

References

[1] G. Caire and S. Shamai (Shitz), "On achievable rates in a multi-antenna broadcast downlink," *in Proceedings of Annual Allerton Conference on Communication, Control and Computing*, Page(s): 1188 - 1193, Monticello, USA, October 2000.

[2] M. Costa, "Writing on dirty paper," *IEEE Transactions on Information Theory*, Vol. 29, No. 3, Page(s): 439 - 441, May 1983.

[3] U. Erez, S. Shamai (Shitz), and R. Zamir, "Capacity and lattice-strategies for cancelling known interference," *Proceedings of International Symposium on Information Theory*, Honolulu, Hawaii, Nov. 2000.

[4] M. Tomlinson, "New automatic equalizer employing modulo arithmetic," *Electronics Letters*, Page(s): 138 - 139, March 1971.

[5] H. Miyakawa and H. Harashima, "A method of code conversion for digital communication channels with intersymbol interference," *Transactions of the Institute of Electronic Communication Engineering, Japan*, Page(s): 272 - 273, June 1969.

[6] W. Yu and J. M. Cioffi, "Trellis precoding for the broadcast channel," *Proceedings of IEEE Globecom*, Page(s): 1344 - 1348, November 2001.

[7] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," *IEEE Transactions on Information Theory*, Vol. 48, Page(s): 1250 - 1276, June 2002.

[8] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of Gaussian multiple-access and broad- cast channels," *Proceedings of International Symposium on Information Theory*, Page(s): 500 - 504, June 2002.

[9] M. Schubert and H. Boche, "Joint dirty-paper pre-coding and downlink beamforming," *IEEE* 7^{th} *International Symposium on Spread-Spectrum Technologies and Applications*, Page(s): 536 - 540, September 2002.

[10] R.F.H. Fischer, "Precoding and Signal Shaping for Digital Transmission," *IEEE, Wiley-Interscience*, 2002.

[11] T.A. Cover and J.A. Thomas, “Elements of Information Theory,” *John Wiley and Sons*, 1991.
[12] G.D. Forney and M.V. Eyuboglu, “Trellis precoding: combined coding, precoding and shaping for intersymbol interference channels,” *IEEE Transactions on Information Theory*, Vol. 5, No. 2, Page(s): 301 - 314, March 1992.
[13] R. A. Monziano and T. W. Miller, *Introduction to Adaptive Arrays*, New York: Wiley, 1980.
[14] B.D. Van Veen and K.M. Buckley, “Beamforming: A versatile approach to spatial filtering,” *IEEE Acoustics, Speech and Signal Processing Magazine*, Vol. 5, No. 2, Page(s): 4 - 24, April 1988.
[15] H. Boche and M. Schubert, “Analysis of different precoding/decoding strategies for multiuser beamforming,” *The 57th IEEE Semiannual VTC 2003-Spring*, Vol. 1, April 22-25, Page(s): 39 - 43, April 2003.

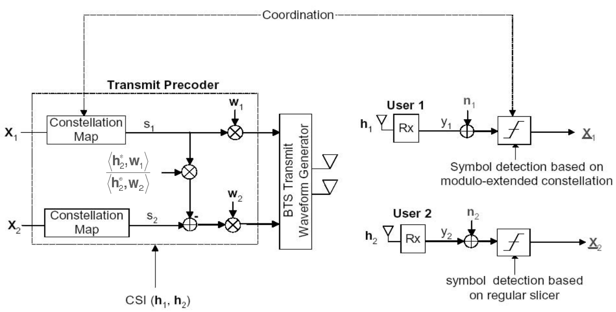

Fig. 1. Downlink broadcast channel and transmit precoding for the two user scenario. Channel knowledge for both users, $\mathbf{h}_1, \mathbf{h}_2$ respectively, is assumed available at the BTS.

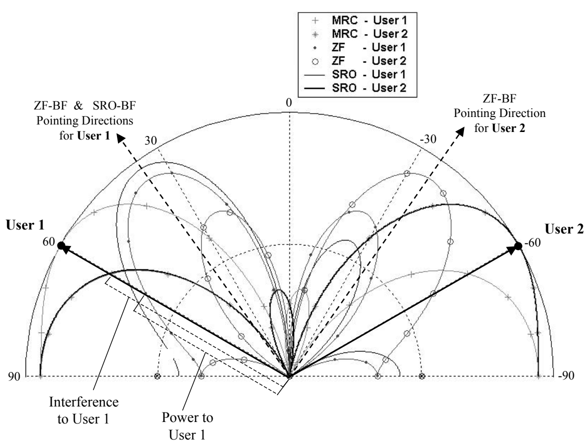

Fig. 2. Beamforming options for $M_t = 3$ and uniform linear array (ULA), with element spacing $\lambda/2$. The figure plots three different beamforming strategies. The two users are located at $+60$ and -60 degrees with respect to the 0 degree reference. The beamforming options considered are: Transmit MRC, ZF, sum-rate optimal.

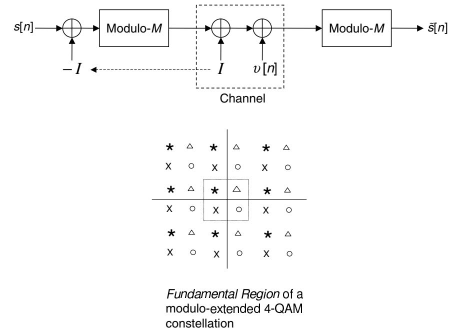

Fig. 3. Illustrating Tomlinson-Harashima Precoding. The transmitter uses uncoded square M-QAM modulation. The transmit symbol $s[n]$ will experience Gaussian noise, $\nu[n]$ and interference I in the channel. If the transmitter has complete non-causal knowledge of the interference, it may be pre-subtracted from $s[n]$ prior to transmission. Since the pre-subtraction may result in a power enhancement, a modulo-M operation is used to restrict signal excursions to the *fundamental* region.

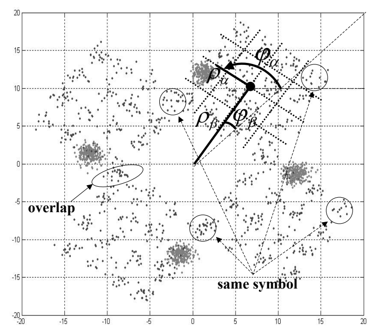

Fig. 4. Illustrating slicer operation. User 1 transmits a symbol s_1 drawn from a 16-QAM constellation and user 2 transmits a symbol s_2 drawn from a 4-QAM constellation. User 2 decodes after compensating for the gain and phase of the effective channel $\sqrt{E_2}\mathbf{h}_2^T\mathbf{w}_2$. User 1 has knowledge of $\rho_\alpha, \rho_\beta, \varphi_\alpha$, and φ_β. The four circles indicate which points from the extended received constellation for user 1 map to the same transmitted symbol. Depending upon the value of ρ_α and ρ_β, some points in the extended received constellation may overlap causing decoding ambiguity. Adaptive constellation switching is used to mitigate this decoding ambiguity.

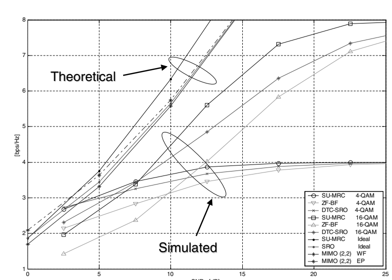

Fig. 5. Sum-Rate Capacity and “Throughput” vs. SNR.

