PERFORMANCE EVALUATION OF 2-ELEMENT ARRAYS OF CIRCULAR PATCH ANTENNAS IN INDOOR CLUSTERED MIMO CHANNELS

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ABSTRACT

In this paper we present a computationally efficient method to evaluate the performance of circular patch arrays (CPAs) in clustered MIMO channel models. The proposed method is based on bounds to the eigenvalues of the spatial correlation matrix that are shown to depend only on the channel angle spread. From these bounds, we derive a closed-form expression of the MIMO ergodic capacity as a function of the angle spread, which is used to estimate the performance of CPAs in different propagation environments. This method yields dramatic reduction in computational complexity due to the reduced number of channel parameters required for performance evaluation of CPAs. Simulation results show that through this method it is possible to predict the performance of CPAs, with negligible capacity loss (below 2%), for the most practical channel scenarios.[1]

INTRODUCTION

The performance of a MIMO communication link depends on the spatial correlation, which is a function of the characteristics of the propagation channel (i.e., angle of arrival/departure, angle spread, number of scatterers) as well as the transmit/receive array parameters [1]. For arrays of uniformly spaced antennas, the throughput that a MIMO channel can support depends on element spacing [2], [3], number of antennas [4], [5], array aperture [6], [7], and array geometry [1], [8], [9]. In typical MIMO systems, however, size constraints often prevent the antennas from being placed far apart (e.g., antenna placement in notebook computers or mobile phones). Therefore, space diversity may not provide the complete solution for next generation wireless handsets and alternative techniques such as polarization/pattern diversity [10]–[13] may be preferable. To exploit pattern diversity the antennas are designed to radiate with orthogonal radiation patterns as a means to create uncorrelated channels across different array elements. In [14] we showed the benefit of pattern over space diversity by analyzing the performance of 2-element arrays of circular patch antennas, or circular patch arrays (CPAs), in spatially correlated MIMO channels. In this contribution we extend that analysis to more realistic channel models for indoor environments, accounting for higher number of channel parameters.

We consider clustered channel models, where the scatterers around the transmit/receive antenna arrays are modelled as *clusters*. There are essentially three parameters that characterize clustered channel models: number of clusters, mean angles of arrival/departure (AoA/AoD) and angle spread (AS). Different channel scenarios are defined by different values of these parameters, as described in the IEEE 802.11n channel model [15] for wireless local area networks (WLANs). Due to the limited number of channel parameters, clustered channel models are particularly suitable for performance analysis of MIMO arrays in realistic propagation scenarios. To ensure MIMO array designs to be robust in a variety of propagation scenarios, the array performance needs to be evaluated exhaustively for different values of the channel parameters. An exhaustive performance evaluation, however, requires high computational complexity and may be intractable for practical designs. In this paper we propose a method to evaluate the performance of 2-element CPAs with a reduced number of channel parameters. This method is based on an approximation that yields minimal error over the predicted MIMO capacity.

We first compute the correlation coefficients of the CPA in a multi-cluster channel model, extending the results we derived in [14] for single-cluster case. From these correlation coefficients we compute bounds to the

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eigenvalues of the spatial correlation matrix and MIMO channel capacity. We express these bounds as a function only of the AS, avoiding the dependence one the other channel parameters. We show that by employing these bounds it is possible to estimate exhaustively the channel capacity in different channel scenarios, with minimal error and reduced computational complexity.

MODEL DESCRIPTION AND ARRAY PERFORMANCE EVALUATION

In this section, we first review some properties of CPAs and clustered MIMO channel models for indoor environments. Then we describe a methodology to evaluate the performance of CPAs in realistic channel environments.

Circular Patch Array (CPA)

The properties of circular microstrip antennas have been studied in [16]–[18]. In [16] it was shown that, by exciting different modes of circular patch antennas, it is possible to obtain different radiation properties. In addition, by varying the size of the antennas as well as the feed location, different polarizations and radiation patterns can be generated in far-field. In this paper we use the orthogonality of the radiation patterns of circular patch antennas as a means to reduce correlation between the diversity branches of the MIMO array.

We express the electric field of n-th mode excited inside the circular patch antenna as a function of its θ and ϕ far-field components as

$$E_\theta^{(n)}(\phi,\theta) = \Gamma(\rho,n)\,(J_{n+1} - J_{n-1})\cos\left[n(\phi-\phi_0)\right] \quad (1)$$

$$E_\phi^{(n)}(\phi,\theta) = -\Gamma(\rho,n)\,(J_{n+1} + J_{n-1})\cos\theta\sin\left[n(\phi-\phi_0)\right] \quad (2)$$

with

$$\Gamma(\rho,n) = e^{jn\pi/2}\frac{V_0^{(n)}}{2}k_0\rho \quad (3)$$

where $V_0^{(n)}$ is the input voltage, k_0 is the wavenumber, $J_n = J_n(k_0\rho\sin\theta)$ is the Bessel function of the second kind and order n, ρ is the radius of the microstrip antenna and ϕ_0 is the reference angle corresponding to the feed point of the antenna [16].

We consider 2-element CPA with collocated antennas stacked on top of each other, as described in [17]. We excite the same mode for both the elements of the MIMO array and tune the phase ϕ_0 to produce orthogonal radiation patterns between the two elements. In particular, we feed one antenna with $\phi_0^{(1)} = 0$ and the other antenna with $\phi_0^{(2)} = \pi/(2n)$.

We assume the angles of arrival/departure are distributed only over the azimuth directions (i.e., $\theta = \pi/2$). Over these directions, the far-field of the circular patch antenna is only vertically polarized and the ϕ component of the far-field in (2) is zero. From equation (1) we derive the array response of the CPA as

$$\mathbf{a}(\phi) = \gamma(\rho,n)\left[\cos(n\phi),\ \ \sin(n\phi)\right]^T \quad (4)$$

where

$$\gamma(\rho,n) = \Gamma(\rho,n)\left[J_{n+1}(k_0\rho) - J_{n-1}(k_0\rho)\right] \quad (5)$$

and ϕ is the azimuth angle of arrival/departure. In our analysis we fix the overall power radiated by the array to be a constant for any mode. In practice, we choose the input voltage $V_0^{(n)}$ to ensure that $|\gamma(\rho,n)|$ is constant across different modes and $||\mathbf{a}||_2^2 = |\gamma(\rho,n)|^2 = N = 2$, where N denotes either the number of transmit (N_t) or receive (N_r) array elements.

Clustered MIMO Channel Model

We consider a narrowband MIMO system, with N_t transmit antennas and N_r receive antennas. We assume zero-mean spatially correlated channels and express the MIMO channel matrix as

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \quad (6)$$

where $\mathbf{R}_t$ and $\mathbf{R}_r$ denote the transmit and receive spatial correlation matrices[2], respectively, and $\mathbf{H}_w \in \mathbb{C}^{N_r\times N_t}$ is the matrix of complex Gaussian fading coefficients. For the duration of this paper, we use $\mathbf{R}$ to refer to either the transmit or receive spatial correlation matrices.

The entries of the spatial correlation matrix ($\mathbf{R}$) are a function of the transmit/receive array and the spatial characteristics of the MIMO channel. In this paper we derive the spatial correlation coefficients (i.e., entries of the matrix $\mathbf{R}$) in closed-form for CPAs and use them to analyze the array performance in clustered channel models. In clustered channel models, the scattering objects around the transmit/receive arrays are modelled as *clusters*. Each cluster is characterized by a mean angle of arrival/departure (AoA/AoD) defined by the solid angle $\Omega_c = (\phi_c,\theta_c)$. In this paper we assume the AoAs/AoDs to be distributed only over the azimuth directions, which is reasonable assumption for the most common channel models [19]. Depending on the system bandwidth, the excess delay across different propagation paths may not be resolvable. In this case, multiple AoAs are defined

[2]We assume the spatial correlation matrices to satisfy the trace constraints $\mathrm{Tr}(\mathbf{R}_t) = N_t$ and $\mathrm{Tr}(\mathbf{R}_r) = N_r$

with an offset ϕ with respect to the mean AoA of the cluster (ϕ_c). This angle of arrival ϕ is generated according to certain probability density function (p.d.f.) that models the power angle spectrum (PAS). The standard deviation (σ_ϕ) of the PAS defines the angle spread (AS) of the channel.

Several distributions have been proposed to approximate the empirically observed PAS. The analysis presented in this paper employs the Laplacian p.d.f., that has been shown to be especially attractive for a variety of outdoor [20] and indoor [21]–[23] scenarios, and has been adopted by the IEEE 802.11n standard channel model [15] for indoor environments.

Performance Evaluation of CPAs in Clustered MIMO Channel Models

There are essentially three parameters that characterize MIMO clustered channel models:

- **Number of clusters** (N_c);
- **Mean AoA** of the clusters: $\mathbf{\Phi} = [\phi_c^{(1)}, \ldots, \phi_c^{(N_c)}]$, with $\phi_c^{(i)} \in \mathcal{A}_\phi$;
- **Angle spread** of the clusters: $\mathbf{\Sigma} = [\sigma_\phi^{(1)}, \ldots, \sigma_\phi^{(N_c)}]$, with $\sigma_\phi^{(i)} \in \mathcal{A}_\sigma$

where $\mathcal{A}_\phi$ and $\mathcal{A}_\sigma$ denote the feasible set of values of $\phi_c^{(i)}$ and $\sigma_\phi^{(i)}$, respectively. For example, the IEEE 802.11n channel model [15] assumes $\mathcal{A}_\phi = \{0^{\rm o}, \ldots, 360^{\rm o}\}$ and $\mathcal{A}_\sigma = \{15^{\rm o}, \ldots, 50^{\rm o}\}$.

In order for CPA designs to be robust in a large variety of channel environments, their performance have to be exhaustively evaluated over a number of channel scenarios defined by

$$\mathcal{N} = N_c|\mathcal{A}_\phi||\mathcal{A}_\sigma| \tag{7}$$

where $|\cdot|$ denotes the cardinality of a set. In practical simulation environments, the cardinality of the sets $\mathcal{A}_\phi$ and $\mathcal{A}_\sigma$ depends on the quantization of the values assigned to the channel parameters. Equation (7) clearly shows that an exhaustive evaluation of the performance of the CPA over the feasible channel scenarios would require high computational complexity. In this paper we propose a method to decrease this complexity by reducing the number of channel parameters required for performance evaluations of the CPA. This method is based on bounds to the eigenvalues of the spatial correlation matrix.

SPATIAL CORRELATION COEFFICIENTS

For a narrowband system with N_c clusters, the spatial correlation between the ℓ-th and m-th antennas of the MIMO array can be expressed as

$$r_{\ell,m} = \frac{1}{N_c}\sum_{i=1}^{N_c} r_{\ell,m}^{(i)} \tag{8}$$

where $r_{\ell,m}^{(i)}$ is the correlation coefficient corresponding to the i-th cluster, and the normalization factor before the summation is to satisfy the trace constraint of the spatial correlation matrix (i.e., $\mathrm{Tr}(\mathbf{R}) = N$).

For the i-th cluster, we express the correlation coefficient across the ℓ-th and m-th antennas of the CPA as [12]

$$r_{\ell,m}^{(i)} = \int_{4\pi} S_i(\Omega)\underline{E}_\ell(\Omega)\underline{E}_m^*(\Omega)d\Omega \tag{9}$$

where $\Omega = (\phi, \theta)$ is the solid angle, $\underline{E}_\ell(\Omega)$ is the far-field of the ℓ-th circular patch, $S_i(\Omega)$ is the PAS of the i-th cluster. For the sake of simplicity, we assume the PAS over the θ angles to be independent from the ϕ angles, which is a good assumption for certain channel environments [19]. Thus $S(\Omega) = S_\phi(\phi)S_\theta(\theta)$. We define the power azimuth spectrum as $S_\phi(\phi) = P(\phi)\delta(\phi - \phi_c)$, where $\delta(\phi)$ is the delta function and ϕ_c is the mean angle of arrival of the cluster. We generate $P(\phi)$ according to the truncated Laplacian distribution defined as [15]

$$P(\phi) = \begin{cases} \frac{1}{\sqrt{2}\sigma_\phi\left(1-e^{-\sqrt{2}\pi/\sigma_\phi}\right)} \cdot e^{-|\sqrt{2}\phi/\sigma_\phi|} & \text{if } \phi \in [-\pi, \pi); \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where σ_ϕ is the standard deviation of the power azimuth spectrum. Since the elevation angle spread is generally small compared to the azimuth spread and the directions of arrival/departure are mostly localized over the azimuth directions [19], we assume $S_\theta(\theta) = \delta(\theta - \pi/2)$ to simplify our analysis.

In [14] we derived closed-form expressions of the correlation coefficients in (9) by using (10), for single cluster case (i.e., $N_c = 1$). The auto- and cross-correlation coefficients are given by (11) and (12), respectively, where $\gamma(\rho, n)$ is defined as in (5).

Now we derive analytically these correlation coefficients considering multiple clusters. To simplify the analysis we assume that the N_c clusters experience the same angle spread (i.e., $\sigma_\phi^{(i)} = \sigma_\phi$, $\forall i = 1, \ldots, N_c$). Under this assumption, substituting (11) and (12) into (8), we write the auto- and cross-correlation coefficients as in (13) and (14), respectively. Note that the auto-correlation coefficient for the second patch is derived from equation (13), accounting for the angle shift ($\phi_0^{(2)}$) across the two antennas, and $r_{2,2}(\phi_c, \sigma_\phi) = r_{1,1}(\phi_c -$

$\phi_0^{(2)}, \sigma_\phi)$. Moreover, the cross-correlation coefficient $r_{2,1}$ has the same expression as (14).

CPA PERFORMANCE ANALYSIS

In this section, we show an expression of the MIMO ergodic capacity as a function of the eigenvalues of the spatial correlation matrix and derive bounds to these eigenvalues. From these bounds we express the MIMO ergodic capacity in closed-form only as a function of the angle spread (σ_ϕ), without any dependency on the number of clusters (N_c) and mean AoA (ϕ_c). This closed-form capacity expression is useful tool to predict the performance of CPAs for the most practical channel scenarios, with minimal error and reduced computational complexity.

MIMO Ergodic Capacity

We consider the tight upper bound to the ergodic capacity for spatial multiplexing (SM) systems (with equal power allocation across the transmit antennas) for spatially correlated channels, reported in [24]. We assume zero-mean single-sided (only at the transmitter) correlated MIMO channels. This upper bound is expressed as

$$C_{\mathrm{ub}} = \log_2 \left[\sum_{k=0}^{n} \left(\frac{\gamma_{\mathrm{o}}}{N_t} \right)^k \frac{N_r!}{(N_r - k)!} \sum_{\alpha_k} |\mathbf{R}_{\alpha_k}^{\alpha_k}| \right] \quad (15)$$

where γ_{o} is the signal-to-noise ratio (SNR), $n = \min(N_r, N_t)$, α_k is an ordered subset of $\{1, ..., n\}$ with measure $|\alpha_k| = k$ and $\mathbf{R}_{\alpha_k}^{\alpha_k}$ denotes the $k \times k$ sub-matrix lying in the α_k rows and α_k columns of the transmit spatial correlation matrix $\mathbf{R}$.

For the case of $N_t = N_r = N = 2$, we get $|\mathbf{R}_{\alpha_0}^{\alpha_0}| = 1$, $|\mathbf{R}_{\alpha_1}^{\alpha_1}| = N = 2$ and $|\mathbf{R}_{\alpha_2}^{\alpha_2}| = |\mathbf{R}| = \lambda_1 \lambda_2$. Then, we express (15) as

$$C_{\mathrm{ub}} = \log_2 \left[1 + 2\,\gamma_{\mathrm{o}} + \frac{\gamma_{\mathrm{o}}^2}{2} \lambda_1 \lambda_2 \right] \quad (16)$$

where λ_1 and λ_2 are the maximum and minimum eigenvalues of the spatial correlation matrix $\mathbf{R}$, respectively.

Eigenvalues of the Spatial Correlation Matrix

The eigenvalues $\lambda_{(1,2)}$ of the 2×2 spatial correlation matrix ($\mathbf{R}$) are computed as

$$\begin{aligned} \lambda_{(1,2)} &= \lambda_{(1,2)}(N_c, \mathbf{\Phi}, \sigma_\phi) \\ &= \tfrac{1}{2} \left[(r_{1,1} + r_{2,2}) \pm \sqrt{4 r_{1,2} r_{2,1} + (r_{1,1} - r_{2,2})^2} \right]. \end{aligned} \quad (17)$$

The eigenvalues in (17) depend on the channel parameters (i.e., N_c, $\mathbf{\Phi}$ and σ_ϕ) through the spatial correlation coefficients in (13) and (14). Hereafter, we derive bounds to these eigenvalues for the CPA, assuming the mode number n to be even. Similar results can be easily derived for n odd, by using similar approximation as in [25].

We expand the two terms in (17) within the brackets by substituting the correlation coefficients in (13) and (14), and derive the expressions (18) and (19). Note that the upper bound in (19) follows from the inequalities $\cos\alpha \leq 1$ and $\sin\alpha \leq 1$, $\forall \alpha \in [0, 2\pi)$. Substituting

$$r_{1,1}^{(i)}(\phi_c^{(i)}, \sigma_\phi^{(i)}) = \frac{|\gamma(\rho,n)|^2}{\left(1 - e^{-\sqrt{2}\pi/\sigma_\phi^{(i)}}\right)} \frac{(n\sigma_\phi^{(i)})^2}{1 + 2(n\sigma_\phi^{(i)})^2} \left[1 - e^{-\sqrt{2}\pi/\sigma_\phi^{(i)}} + \frac{1 - e^{-\sqrt{2}\pi/\sigma_\phi^{(i)}} \cos(n\pi)}{(n\sigma_\phi^{(i)})^2} \cos^2(n\phi_c^{(i)}) \right] \quad (11)$$

$$r_{1,2}^{(i)}(\phi_c^{(i)}, \sigma_\phi^{(i)}) = \frac{|\gamma(\rho,n)|^2}{2\left[1 + 2(n\sigma_\phi^{(i)})^2\right]} \sin(2n\phi_c^{(i)}) \quad (12)$$

$$r_{1,1}(N_c, \mathbf{\Phi}, \sigma_\phi) = \frac{|\gamma(\rho,n)|^2}{\left(1 - e^{-\sqrt{2}\pi/\sigma_\phi}\right)} \frac{(n\sigma_\phi)^2}{1 + 2(n\sigma_\phi)^2} \left[1 - e^{-\sqrt{2}\pi/\sigma_\phi} + \frac{1 - e^{-\sqrt{2}\pi/\sigma_\phi} \cos(n\pi)}{N_c (n\sigma_\phi)^2} \sum_{i=1}^{N_c} \cos^2(n\phi_c^{(i)}) \right] \quad (13)$$

$$r_{1,2}(N_c, \mathbf{\Phi}, \sigma_\phi) = \frac{|\gamma(\rho,n)|^2}{2N_c\left[1 + 2(n\sigma_\phi)^2\right]} \sum_{i=1}^{N_c} \sin(2n\phi_c^{(i)}) \quad (14)$$

(18) and (19) into (17) we derive the bounds to the eigenvalues of the CPA as

$$\lambda_1(\sigma_\phi) \leq 1 + \frac{1}{1+2(n\sigma_\phi)^2} \tag{20}$$

$$\lambda_2(\sigma_\phi) \geq 1 - \frac{1}{1+2(n\sigma_\phi)^2}. \tag{21}$$

Computational Efficient Method to Evaluate the Performance of CPAs

We observe that the eigenvalue bounds in (20) and (21) depend only on the angle spread σ_ϕ (as opposed to the exact eigenvalues in (17) that depend also on N_c and $\mathbf{\Phi}$). Substituting (20) and (21) in (16) we derive the following bound to the MIMO ergodic capacity for the CPA, expressed only as a function of the AS [3]

$$C_{\rm ub} \geq \log_2 \left[1 + 2\,\gamma_{\rm o} + \frac{\gamma_{\rm o}^2}{2} \left(1 - \frac{1}{(1+2(n\sigma_\phi)^2)^2} \right) \right]. \tag{22}$$

Hence, to evaluate the channel capacity due to the CPA it is sufficient to compute the array performance only as a function of the AS, over a reduced number of channel scenarios given by

$$\mathcal{N} = |\mathcal{A}_\sigma|. \tag{23}$$

By comparing (23) against (7) it is possible to appreciate the dramatic reduction in computational complexity provided by the proposed method.

[3]We define $\lambda_1 \geq 1$ and $\lambda_2 \leq 1$, according to the trace constraint $\mathrm{Tr}(\mathbf{R}) = N = 2$

SIMULATION RESULTS

Hereafter, we present simulation results to evaluate the error due to the bounds in (20), (21) and (22). These results assume the antennas of the CPA to be excited with mode number 4, that yields good size/performance tradeoff as discussed in [14].

Fig. 1 compares the exact eigenvalue λ_2 in (17) against the lower bound in (21). The channel is simulated with $N_c = 2$, mean AoAs generated in the range $[0, 180^{\rm o}]$ and $\sigma_\phi = 20^{\rm o}$. It is possible to see that the lower bound is

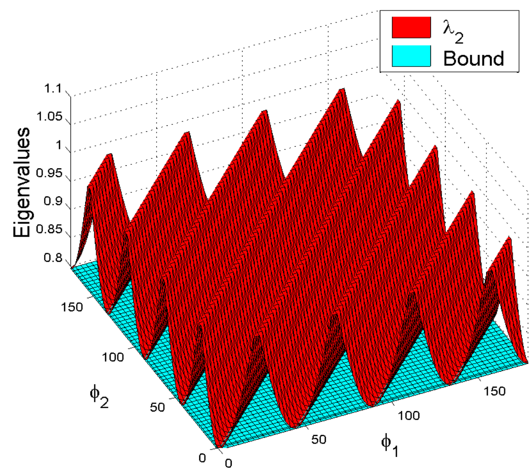

Fig. 1. Exact eigenvalue (λ_2) and lower bound in (21) as a function of the mean AoAs ϕ_1 and ϕ_2. The channel is simulated with $N_c = 2$, mean AoAs generated in the range $[0, 180^{\rm o}]$ and $\sigma_\phi = 20^{\rm o}$.

$$\begin{aligned} r_{1,1} + r_{2,2} &= \frac{|\gamma(\rho,n)|^2}{\left(1-e^{-\sqrt{2}\pi/\sigma_\phi}\right)} \frac{(n\sigma_\phi)^2}{1+2(n\sigma_\phi)^2} \\ &\times \left[2\left(1 - e^{-\sqrt{2}\pi/\sigma_\phi}\right) + \frac{1-e^{-\sqrt{2}\pi/\sigma_\phi}\cos(n\pi)}{N_c(n\sigma_\phi)^2} \sum_{i=1}^{N_c} \left[\cos^2(n\phi_c^{(i)}) + \sin^2(n\phi_c^{(i)})\right] \right] \\ &= \frac{|\gamma(\rho,n)|^2}{\left(1-e^{-\sqrt{2}\pi/\sigma_\phi}\right)} \frac{(n\sigma_\phi)^2}{1+2(n\sigma_\phi)^2} \left[2\left(1 - e^{-\sqrt{2}\pi/\sigma_\phi}\right) + \frac{1-e^{-\sqrt{2}\pi/\sigma_\phi}\cos(n\pi)}{(n\sigma_\phi)^2} \right] \\ &\overset{(n\ \text{even})}{=} |\gamma(\rho,n)|^2 \end{aligned} \tag{18}$$

$$\begin{aligned} 4r_{1,2}r_{2,1} + (r_{1,1} - r_{2,2})^2 &\overset{(n\ \text{even})}{=} \left[\frac{|\gamma(\rho,n)|^2}{1+2(n\sigma_\phi)^2}\right]^2 \left\{ \left[\frac{1}{N_c}\sum_{i=1}^{N_c}\cos(2n\phi_c^{(i)})\right]^2 + \left[\frac{1}{N_c}\sum_{i=1}^{N_c}\sin(2n\phi_c^{(i)})\right]^2 \right\} \\ &\leq \left[\frac{|\gamma(\rho,n)|^2}{1+2(n\sigma_\phi)^2}\right]^2 \end{aligned} \tag{19}$$

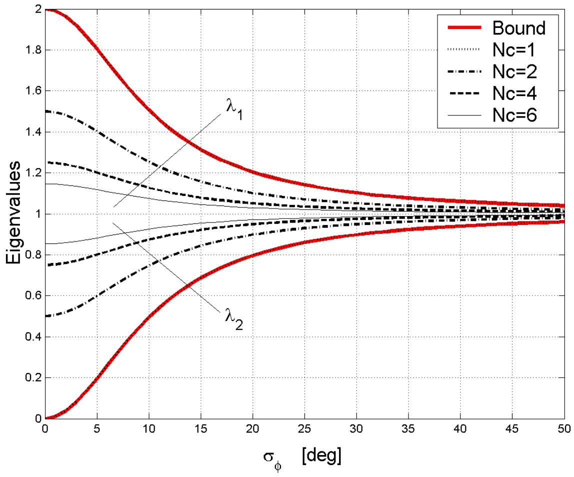

Fig. 2. Exact eigenvalues (λ_1 and λ_2) and bounds in (20) and (21). The channel is simulated with variable N_c, mean AoAs generated as $\phi_c^{(i)} = \tilde{\phi}(i-1)/N_c$ (with $i = 1, \ldots, N_c$ and $\tilde{\phi} = 120^{\rm o}$) and variable AS ($\sigma_\phi$).

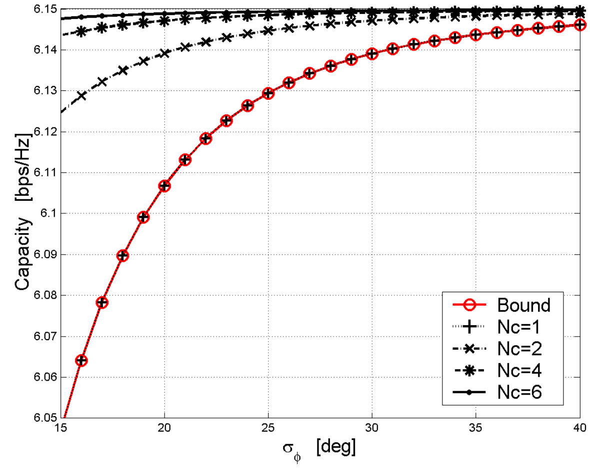

Fig. 3. Ergodic capacity in (16) and bound in (22), with $\gamma_{\rm o} = 10$ dB. The channel is simulated with variable N_c, mean AoAs generated as $\phi_c^{(i)} = \tilde{\phi}(i-1)/N_c$ (with $i = 1, \ldots, N_c$ and $\tilde{\phi} = 120^{\rm o}$) and variable AS ($\sigma_\phi$).

closed to the exact expression of λ_2 for any combination of values of ϕ_1 and ϕ_2.

In Fig. 2 we compare the bounds to λ_1 and λ_2 against their exact expressions as a function of the cluster AS and for different values of N_c. The mean AoAs are generated as $\phi_c^{(i)} = \tilde{\phi}(i-1)/N_c$ (with $i = 1, \ldots, N_c$ and $\tilde{\phi} = 120^{\rm o}$) and variable AS ($\sigma_\phi$). Fig. 2 shows that the bounds in (20) and (21) are close to the exact expression of the eigenvalues in (17) for $\sigma_\phi > 15^{\rm o}$. This value corresponds to the lowest AS defined in the IEEE 802.11n channel model [15], which makes the proposed method practical for indoor environments in the context of WLANs.

In Fig. 3 we compare the capacity "lower bound" ($C_{\rm lb}$) in (22) against the "exact capacity" ($C_{\rm exct}$) obtained by substituting the exact eigenvalues in (17) into (16), with $\gamma_{\rm o} = 10$ dB. Fig. 3 shows that, for the most common values of AS (i.e., $\sigma_\phi > 15^{\rm o}$), $C_{\rm lb}$ closely approximates $C_{\rm exct}$.

Next, we quantify the capacity loss due to the eigenvalue bounds in (20) and (21). We define the capacity loss as

$$C_{\rm loss} = \frac{C_{\rm exct} - C_{\rm lb}}{C_{\rm exct}}. \qquad (24)$$

In Fig. 4 the channel is simulated with $N_c = 2$, mean AoAs generated in the range $[0, 180^{\rm o}]$, $\sigma_\phi = 15^{\rm o}$ and $\gamma_{\rm o} = 10$ dB. Fig. 4 shows that the capacity loss due to the eigenvalue bounds is below 2% for any combination of values of ϕ_1 and ϕ_2.

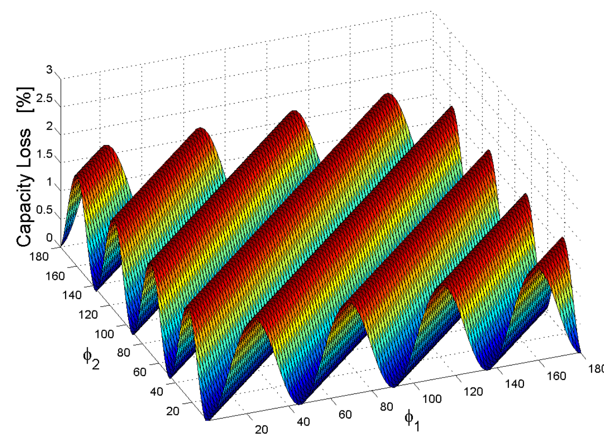

Fig. 4. Ergodic capacity loss (in percentage) due to the eigenvalue bounds in (20) and (21), as a function of the mean AoAs ϕ_1 and ϕ_2, with $\gamma_{\rm o} = 10$ dB. The channel is simulated with $N_c = 2$, mean AoAs generated in the range $[0, 180^{\rm o}]$ and $\sigma_\phi = 15^{\rm o}$.

These results suggest that to predict the performance of the CPA in a variety of channel environments it is sufficient to compute the ergodic capacity only as a function of the AS. The capacity loss ($C_{\rm loss}$) produced by the bound in (22) remains below an acceptable threshold of 2% for the most common channel scenarios.

CONCLUSIONS

We presented a method to reduce the computational complexity for performance evaluations of circular patch arrays (CPAs) in clustered MIMO channel models. This method is based on bounds to the eigenvalues of the spatial correlation matrix and MIMO channel capacity expressed as a function of a single channel parameter: the angle spread. These bounds are used to predict the capacity of the CPA in different propagation environments, avoiding exhaustive performance evaluations over many channel scenarios. We showed that the capacity loss due to these bounds is negligible (below 2%) for the most practical channel scenarios.

These results can be directly applied to the design of CPAs, where a joint optimization of microwave theory and communication theoretic metrics is required. By employing the proposed method it is possible to reduce significantly the complexity of optimization algorithms conceived to enhance the performance of CPA designs in clustered MIMO channel models, which is object of our ongoing investigations.

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