Capacity of Opportunistic Space Division Multiple Access with Beam Selection

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Abstract— **In this paper, a novel transmission technique for the multiple-input multiple-output (MIMO) broadcast channel is proposed that allows simultaneous transmission to multiple users under a limited feedback requirement. During a training phase, the base station modulates a training sequence on multiple sets of randomly generated orthogonal beamforming vectors. Then, based on the users' feedback, the base station opportunistically selects the users and corresponding orthogonal vectors that maximize the sum capacity. From theoretical analysis, the optimal amount of training to maximize the sum capacity is derived as a function of the system parameters. The main advantage of the proposed system is that it provides throughput gains for the MIMO broadcast channel with a small feedback overhead, and provides these gains even with a small number of active users. Numerical simulations show that a 20% gain in sum capacity is achieved (for a small number of users) over conventional opportunistic space division multiple access, and a 100% gain (for a large number of users) over conventional opportunistic beamforming.**

I. Introduction

Multiple-input multiple-output (MIMO) technology can increase throughput in cellular systems by exploiting the spatial dimensions of the MIMO broadcast channel (MIMO-BC) to support transmissions to multiple users. The theoretical sum capacity achievable over the MIMO-BC has been recently analyzed in [1–4]. Practical transmission techniques for the MIMO-BC have been proposed that use beamforming [5–8] or block diagonalization [9–12] to transmit parallel data streams to different users, but still require complete channel state information (CSI) for each user at the transmitter. Unfortunately, systems employing full CSI are limited by large feedback overhead and affected by severe performance degradation due to channel estimation errors. Alternative solutions use only partial CSI and exploit the opportunism inherent in multiuser communication systems [13, 14]. Perhaps the best known approach is opportunistic beamforming (OBF) [13–16].

With OBF, the base station randomly selects a beam for transmission and uses it to send a training sequence. The users send back their signal-to-noise ratio (SNR) corresponding to this beam and the base station schedules the user with the highest SNR (or uses another scheduling rule) for transmission. OBF approaches the performance of the optimal beamforming strategy asymptotically with the number of users [13]. When there are only a few users, the enhanced OBF scheme in [17] employs minislots to send multiple beams and selects the optimal beam for data transmission. OBF-S converges faster to the optimal beamforming strategy due to beam selection. Both OBF and OBF-S, however, do not take full advantage of the multiple dimensions of the MIMO-BC. A solution is to multiplex parallel data streams to different users via opportunistic space division multiple access (OSDMA) [18–20] where orthogonal beams for multiple users are selected to support the best users according to certain scheduling criterion based on the received signal-to-interference-plus-noise ratio (SINR). As with OBF, this approach requires a large number of users to approach full-CSI capacity gains.

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In this paper we present a novel transmission technique for the MIMO-BC based on the concept of OSDMA. The key idea of our approach is to randomly generate multiple sets of orthonormal weight vectors during the training period. Each user responds with their best beamforming weight and SINR for that beamforming vector in the corresponding orthogonal set. Then the base station selects the set of users and beamforming weights, i.e., *beam selection*, that maximizes the sum capacity. We call this scheme OSDMA with beam selection (OSDMA-S). Our technique generalizes OBF-S [17] by enabling parallel data transmissions to multiple users and improves on [18] by using beam selection to reap capacity benefits for a low number of users in the system.

Our main analytical contribution is a theoretical sum capacity expression for OSDMA-S accounting for the penalty due to training. From these expressions we compute the number of training symbols needed for the beam selection at the transmitter by maximizing a bound on the sum capacity with training. Using Monte Carlo simulations we show that about 100% gain in sum capacity is achieved over OBF and OBF-S [13, 17], when there are about fifty active users, thanks to our ability to spatially multiplex information to multiple users. We also show our scheme outperforms conventional OSDMA in [18] (due to beam selection) for relatively low numbers of users and with minimal increased feedback overhead. For example, we show that OSDMA-S yields sum capacity gains higher than 20% over conventional OSDMA when there are less than five active users.

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II. THE PROPOSED OSDMA WITH BEAM SELECTION

A. System Model and Framing Structure

We consider the MIMO broadcast channel with N_t transmit antennas and U users, each with a single receive antenna. Different transmission techniques can be used for the MIMO broadcast channel, but we focus on OSDMA schemes due to the dramatic reduction of feedback information required at the base station. The block diagram of a typical OSDMA system is given in Fig. 1, in which the centralized transmitter constructs $B(\leq N_t)$ random orthonormal beams and sends B pilot symbols to different users through these beams. Each user feeds back the best pilot symbol index and corresponding SINR level among B pilot symbols to the transmitter. Then a subset of users is selected for data transmission, according to certain scheduling criterion, based on the SINR feedback information. OSDMA can effectively exploit multiuser diversity via user selection, and provide multiplexing gain by transmitting parallel data streams over orthonormal beams [18], but the performance is limited if there are not a sufficient number of users.

To effectively amplify the gain due to multiuser diversity (especially when the number of users in the system is low), we propose a new OSDMA scheme employing beam selection (OSDMA-S). In our proposed scheme, the base station constructs multiple sets of B orthonormal beamforming vectors during a training period. A method to construct random sets of orthonormal beams is given in [21]. Then, the set of beams that maximizes the sum capacity are selected, and parallel data streams are transmitted over the B orthogonal beams to different users. More details on the criterion used to select the optimal set of beams and users are provided in the next subsection. In practice, we assume that every time slot of length L has a training period consisting of M *mini-slots* of length τ each. The maximum value of M is $\lfloor \frac{L}{\tau} \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function. That is, τM duration is used for training and $L - \tau M$ duration is used for data transmission in every time slot. The case of $M = 1$ corresponds to the conventional OSDMA or OBF without training. In every mini-slot, B random orthonormal vectors $\left\{\mathbf{v}_{b,m} \in \mathbb{C}^{N_t \times 1},\ m = 1, \ldots, M\right\}$ are generated, where the subscripts b and m denote the beam index and the mini-slot index, respectively. Then, B pilot symbols $\{s_b\}$ are multiplied by the respective beamforming vectors $\mathbf{v}_{b,m}$ and transmitted over the wireless link at every mini-slot ($m = 1, \cdots, M$).

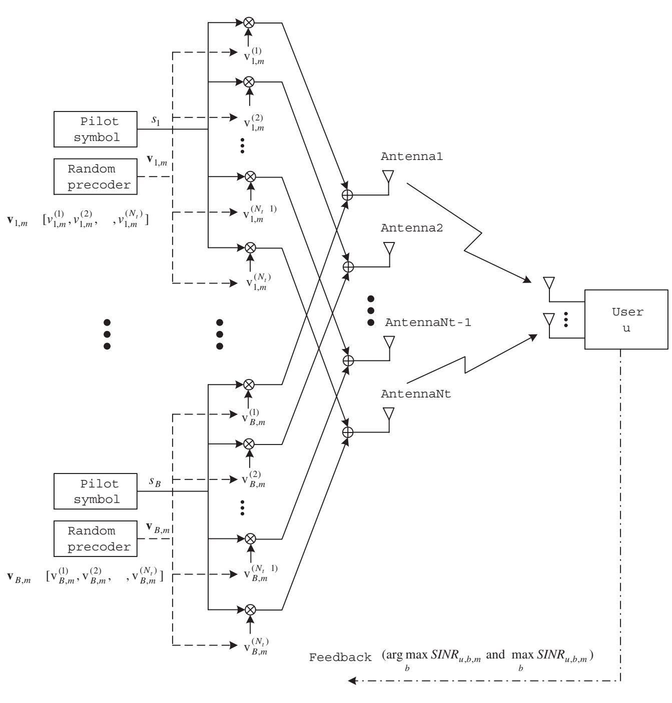

Fig. 1. Block diagram of the proposed system. In every mini-slot, B random beams are generated and transmitted.

If we assume the channels for all the users are static during a time slot, the signal received at user u in the m-th mini-slot is given by

$$y_{u,m} = \sum_{b=1}^{B} \mathbf{h}_u^T \mathbf{v}_{b,m} s_b + n_u \quad \text{for } u = 1, \cdots, U \tag{1}$$

where $\mathbf{h}_u \in \mathbb{C}^{N_t \times 1}$ is the channel vector for the u-th user with independent identically distributed (i.i.d.) complex Gaussian entries $\sim CN(0,1)$, $\mathbf{v}_{b,m} \in \mathbb{C}^{N_t \times 1}$ is the b-th random beamforming vector at the m-th mini-slot, s_b is the b-th transmit pilot symbol, and $n_u \in \mathbb{C}^1$ is the complex zero-mean additive Gaussian noise vector with variance N_o. The superscript T denotes the vector transpose. Moreover, we assume $\mathbb{E}[|s_m|^2] = P/B$ such that the total transmit power is P.

B. Algorithm Description

In every mini-slot, the u-th user computes the following B values of SINR by assuming that s_b is the desired pilot signal (known both at the transmitter and receiver) while the other s_i's are treated as interference as

$$\mathrm{SINR}_{u,b,m} = \frac{|\mathbf{h}_u^T \mathbf{v}_{b,m}|^2}{1/\rho + \sum_{l \neq b} |\mathbf{h}_u^T \mathbf{v}_{l,m}|^2} \tag{2}$$

where $\rho = P/N_o$ is the input SNR. Note that the random variables $\mathrm{SINR}_{u,b,m}$ are i.i.d. for $u = 1, \cdots, U$, whereas for $b = 1, \cdots, B$ and for $m = 1, \cdots, M$ are identically distributed but not independent. We assume, however, that $\mathrm{SINR}_{u,b,m}$ are i.i.d. in b and m as well for analytical tractability. The validity of this assumption is later justified by simulations as in [17].

For every mini-slot, each user feeds back its maximum SINR (i.e., $\max_{1 \leq b \leq B} \mathrm{SINR}_{u,b,m}$ for a given mini-slot m) along with the index b in which the SINR is maximized. The transmitter computes the throughput R_m for all mini-slots as

$$R_m \triangleq \sum_{b=1}^{B} \log \left(1 + \max_{1 \leq u \leq U} \mathrm{SINR}_{u,b,m}\right) \text{ for } m = 1, \cdots, M. \tag{3}$$

Even though there is a small possibility that user u is the strongest for more than one signal s_b, we neglect this small probability because this is very unlikely as U increases. Therefore, we ignore the probability that user u has the maximum

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SINR for different b. If this is the case, the transmitter chooses another user with the second largest SINR user so that B different users are assigned in every slot.

The base station determines a subset of B users (and corresponding beams allocated to those users) based on R_m in (3) for every mini-slot. After one training period (consisting of M mini-slots) the transmitter has M candidate sets of B users (and corresponding beams allocated to those users). Then, the transmitter selects the subset of users (and corresponding beams) with the largest R_m among the M candidates, according to maximum rate scheduling policy [22–24]. Let the required bits for quantizing R_m be Q, the total amount of required feedback per user becomes $M(\log_2 B + Q)$ bits. Finally, parallel data streams are transmitted over the current time slot through the B selected beams, and the resulting throughput is given by

$$R \triangleq (L - \tau M) \max_{1 \le m \le M} R_m \tag{4}$$

where τM is the total overhead due to training. Note that, alternatively, different scheduling criteria can be applied for this user selection and we will consider them in future work.

III. Information Theoretic Capacity of OSDMA with Beam Selection

Hereafter, we derive an analytical expression for the sum-rate capacity in (4) that we use to determine the optimum number of mini-slots M, in the sense of maximum average throughput, as a function of the SINR, number of users, number of transmit antennas and training overhead. Although a long training period (i.e., large M) can increase the multiuser diversity gain, it reduces throughput due to a large training overhead. Therefore, the optimal value of M has to be computed to maximize average throughput as

$$M^{\text{opt}} = \arg \max_{m=1,\cdots,\lfloor \frac{L}{\tau} \rfloor} \mathbb{E}[R]. \tag{5}$$

Because it is hard to obtain a closed form expression of $\mathbb{E}[R]$, we consider an upper bound on $\mathbb{E}[R]$ expressed in terms of M to find the optimum value of M. Using order statistic theory on an upper bound on the maximum random variable [25, pp. 62, Eq.(4.2.6)], the following upper bound on $\mathbb{E}[\max_{1 \le q \le Q} R_q]$ can be obtained by

$$\mathbb{E}\left[\max_{1 \le m \le M} R_m\right] \le \mathbb{E}[R_m] + \frac{M-1}{\sqrt{2M-1}}\sqrt{\text{Var}[R_m]} \tag{6}$$

where R_m is defined in (3) and $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$ can be computed with the probability density function (p.d.f.) of $\max_{1 \le u \le U} \text{SINR}_{u,b,m}$. Since $\mathbf{h}_u$ is an i.i.d. complex Gaussian channel vector, the probability density function $f_s(x)$ of $\text{SINR}_{u,b,m}$ is given by [18]

$$f_s(x) = \frac{e^{-x/\rho}}{(1+x)^B}\left(\frac{1}{\rho}(1+x) + B - 1\right) \tag{7}$$

and the cumulative density function $F_s(x)$ is

$$F_s(x) = 1 - \frac{e^{-x/\rho}}{(1+x)^{B-1}}. \tag{8}$$

Therefore, the p.d.f. of $\max_{1 \le u \le U} \text{SINR}_{u,b,m}$ is $f_{\max}(x) = U f_s(x) F_s^{U-1}(x)$, and $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$ are computed as [18]

$$\mathbb{E}[R_m] = B \int_0^\infty \log(1+x) U f_s(x) F_s^{U-1}(x) dx \tag{9}$$

$$\text{Var}[R_m] = B \left[\int_0^\infty (\log(1+x))^2 U f_s(x) F_s^{U-1}(x) dx - \left(\int_0^\infty \log(1+x) U f_s(x) F_s^{U-1}(x) dx\right)^2\right] \tag{10}$$

Unfortunately, it is not possible to derive closed form expressions of (9) and (10), and we rely on numerical integrations to compute $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$.

Based on (6), we derive the following upper bound on the average sum capacity

$$\mathbb{E}[R] \le (L - \tau M)\left[\mathbb{E}[R_m] + \frac{M-1}{\sqrt{2M-1}}\sqrt{\text{Var}[R_m]}\right] \tag{11}$$

and substituting (11) into (5) we derive the optimal value of M as

$$M^{\text{opt}} = \arg \max_{M=1,\cdots,\lfloor \frac{L}{\tau} \rfloor} \left[\mathbb{E}[R_m] + \frac{M-1}{\sqrt{2M-1}}\sqrt{\text{Var}[R_m]}\right] \tag{12}$$

where we note that $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$ are not dependent on M. Hereafter, we show numerically that the value of M corresponding to the maximum of $\mathbb{E}[R]$ is the same as for the maximum of the upper bound in (11). Hence, (12) is in fact the optimal value of M. Fig. 2 shows the normalized average sum capacity versus the number of mini-slots M used for training, for different numbers of users (U). The average sum capacity through Monte Carlo simulations and the upper bound in (11) are compared. These results assume SNR=10dB, $\frac{\tau}{L} = 5\%$ and $B = N_t = 4$. Note that the upper bound follows closely the empirical curves, especially for large number of users and low values of M. Moreover, for a given U, the capacity increases up to certain value of M due to the benefit of beam selection, and decreases afterwards due to the increasing training overhead. We observe that the optimal number of mini-slots (M^{opt}), corresponding to the maximum value of capacity, is the same for both the empirical and numerical curves, for any number of users. Therefore, it is possible to use (12) to compute M^{opt}, reducing the computational complexity of the performance evaluation of our proposed OSDMA scheme. In Fig. 2, we observe that the value of M^{opt} decreases as the number of users increases due to the larger gain offered by multiuser diversity against beam selection. Moreover, $M^{\text{opt}} \le 5$ for any number of user, revealing that the capacity gains of OSDMA with beam selection are available for a small number of training mini-slots (i.e., a small amount of feedback for the SINR information).

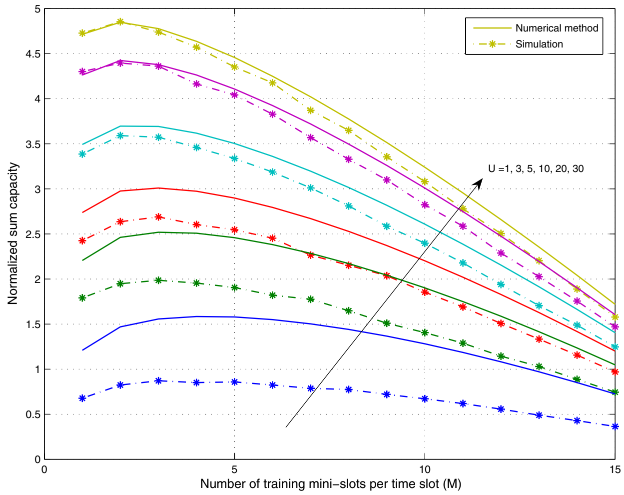

Fig. 2. Normalized throughput obtained through simulations and numerical method, assuming SNR=10 dB, $\frac{\tau}{L} = 5\%$ and $B = N_t = 4$.

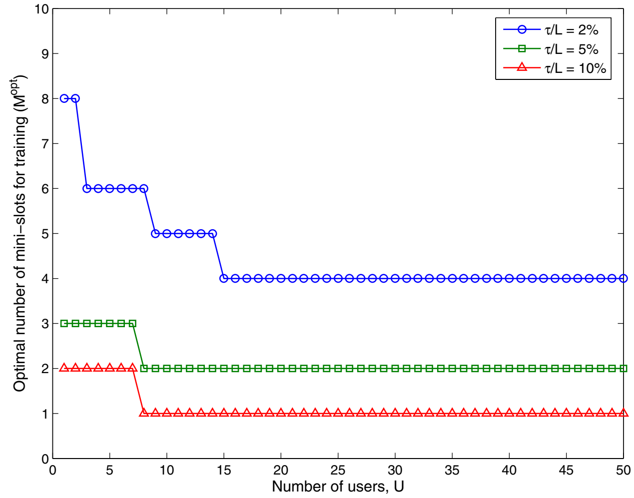

Fig. 3. Optimal number of mini-slots (M^{opt}) for OSDMA versus U, for different normalized lengths of mini-slot (τ/L), with $B = N_t = 4$.

IV. Performance Results

A. Optimal Number of Mini-Slots for Training (M^{opt}) in the Proposed OSDMA-S

We evaluate the optimal number of mini-slots used for training as a function of the number of users (U), SNR, number of transmit antennas ($N_t = B$) and length of the mini-slots (τ/L) through simulation. It should be noted, however, that the same value of M^{opt} can also be obtained through the analytical results in Section III.

In Fig. 3, the values of M^{opt} are plotted as a function of U for different normalized mini-slot lengths (τ/L), with $B = N_t = 4$. The value of M^{opt} decreases for increasing number of users (as for the previous results) and the length of the mini-slots. We observe that for a fixed number of users, different values of τ/L yield similar values of percentage overhead ($M\tau/L$) due to training. For example, when 10 users are in the system, all three values of mini-slot length in Fig. 3 produce the same training overhead $M^{\text{opt}}\tau/L = 10\%$. Moreover, higher training overhead (i.e., $M^{\text{opt}}\tau/L \sim 15\%$) is required for lower number of users (i.e., $U < 5$) to achieve the maximum sum capacity, due to lack of multi-user diversity gain.

B. Performance Comparison With Conventional Schemes

In Fig. 4 we compare OSDMA-S versus OBF-S with $M = M^{\text{opt}}$ as in [17], for different values of N_t, $\tau/L = 5\%$ and $SNR = 5$ dB. In this figure, OSDMA-S outperforms OBF-S especially for large number of users, due to the multiplexing gain of SDMA. We also observe that OBF-S with $M = M^{\text{opt}}$ yields higher sum capacity than OBF with $M = 1$ because of higher degrees of freedom in the selection of the optimal beamforming weights. This gain, however, is not enough to reach the multiplexing gain of OSDMA-S.

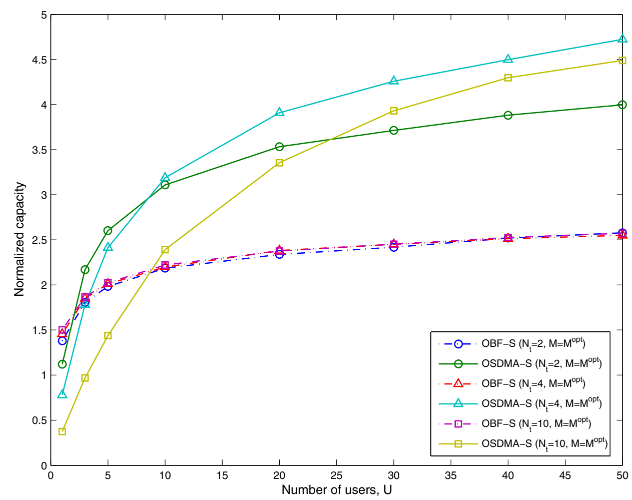

Fig. 4. Normalized sum capacity of OSDMA (with $M = M^{\text{opt}}$ and $B = N_t$) and OBF (with $M = M^{\text{opt}}$ and $M = 1$) as a function of the number of users for different values of N_t, with SNR=5 dB and $\frac{\tau}{L} = 5\%$.

Finally, we compare the performance of OSDMA with and without beam selection. For the case of no beam selection, we assume $M = 1$ and $B = N_t$ as in [18]. Fig. 5 depicts the capacity of different OBF and OSDMA schemes as a function of the number of users. We assume $N_t = 4$, $\tau/L = 2\%$ and $SNR = 5$ dB. We observe that OSDMA-S always outperforms conventional OSDMA for any value of U. Although the gain of OSDMA-S over conventional OSDMA is almost constant, the gain of our method corresponds to 20% gain in sum capacity over conventional OSDMA for $U = 5$, while this gain is reduced to 5% for $U = 50$. For increasing number of users, the relative performance of our proposed

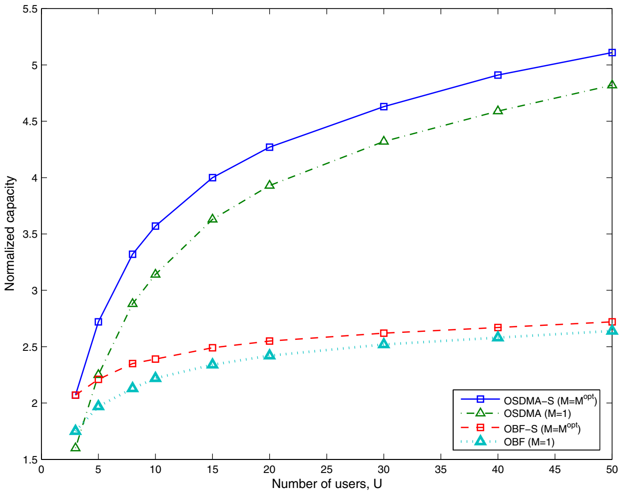

Fig. 5. Performance comparison of OBF and OSDMA schemes with and without beam selection as a function of the number of users U, with SNR=5 dB, $\frac{\tau}{L} = 2\%$ and $B = N_t = 4$.

scheme converges to conventional OSDMA, since the gain due to multiuser diversity becomes dominant compared to the gain of beam selection. Note that the gains achievable through beam selection come at the expense of the increased feedback required to convey the SINR information from the users to the base station during the training period. As an example, for the case of 50 users, the capacity gain shown in Fig. 5 is obtained at cost of $M^{\rm opt} = 4$ SINR feedbacks per user according to Fig. 3. Note that the system parameters τ/L and N_t can be adjusted to maximize the sum capacity for a given constraint on the feedback overhead.

Fig. 5 also shows the capacity of different OBF techniques. We observe that OSDMA-S provides significant capacity gains over OBF, especially for a large number of users, due to SDMA. Interestingly, for $U = 3$, our proposed technique has the same performance as OBF-S due to the adverse effect of the interference, while yielding 20% and 30% gains over conventional OBF and OSDMA, respectively. Table I provides an overall comparison among OBF, OBF-S, OSDMA, and OSDMA-S.

V. Conclusions

We presented a novel transmission scheme for the MIMO broadcast channel that combines OSDMA with beam selection (OSDMA-S). The proposed OSDMA-S provides throughput gains with a small feedback overhead, even for a few active users and generalizes both conventional OBF and OSDMA. We first analyzed the theoretical sum capacity of our scheme, accounting for the penalty due to training overhead. Then, we showed that our schemes achieves up to 20% capacity gains over conventional OSDMA, and a 100% gain over conventional OBF.

References

[1] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Info. Th.*, vol. 49, pp. 1691–1706, July 2003.

[2] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Info. Th.*, vol. 49, pp. 1912–1921, Aug. 2003.

[3] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Info. Th.*, vol. 49, pp. 2658–2668, Oct. 2003.

[4] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," *IEEE Trans. Info. Th.*, vol. 50, pp. 1875–1892, Sep. 2004.

[5] K.-K. Wong, R. D. Murch, and K. B. Letaief, "Performance enhancement of multiuser MIMO wireless communication systems," *IEEE Trans. Comm.*, vol. 50, pp. 1960–1970, Dec. 2002.

[6] S. Serbetli and A. Yener, "Transceiver optimization for multiuser MIMO systems," *IEEE Trans. Sig. Proc.*, vol. 52, pp. 214–226, Jan. 2004.

[7] G. Dimic and N. D. Sidropoulos, "Low-complexity downlink beamforming for maximum sum capacity," *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, vol. 4, pp. 701–704, May 2004.

[8] M. Airy, A. Forenza, R. W. Heath, and S. Shakkottai, "Practical Costa precoding for the multiple antenna broadcast channel," *Proc. IEEE Glob. Telecom. Conf.*, vol. 6, pp. 3942–3946, Nov. 2004.

[9] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Sig. Proc.*, vol. 52, pp. 461–471, Feb. 2004.

[10] K. K. Wong, R. D. Murch, and K. B. Letaief, "A joint-channel diagonalization for multiuser MIMO antenna systems," *IEEE Trans. Wireless Comm.*, vol. 2, pp. 773–786, Jul 2003.

[11] Z. Shen, J. G. Andrews, R. W. Heath, and B. L. Evans, "Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization," *accepted for publication in IEEE Trans. Sig. Proc.*, Sep. 2005.

[12] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, and B. L. Evans, "Sum capacity of multiuser MIMO broadcast channels with block diagonalization," *submitted to IEEE Trans. Wireless Comm.*, Oct. 2005.

[13] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Info. Th.*, vol. 48, pp. 1277–1294, June 2002.

[14] N. Sharma and L. H. Ozarow, "A study of opportunism for multiple-antenna systems," *IEEE Trans. Info. Th.*, vol. 51, pp. 1804–1814, May 2005.

[15] R. Laroia, J. Li, S. Rangan, and M. Srinivasan, "Enhanced opportunistic beamforming," *Proc. IEEE Veh. Technol. Conf.*, vol. 3, pp. 1762–1766, Oct. 2003.

[16] L. Zan and S. A. Jafar, "Combined opportunistic beamforming and receive antenna selection," *Proc. IEEE Wireless Comm. and Net. Conf.*, vol. 2, pp. 1007–1011, Mar. 2005.

[17] I. Kim, S. Hong, S. Chassemzadeh, and V. Tarokh, "Optimum opportunistic beamforming based on multiple weighting vectors," *Proc. IEEE Int. Conf. on Comm.*, pp. 2427–2430, May 2005.

[18] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channel with partial side information," *IEEE Trans. Info. Th.*, vol. 51, pp. 506–522, Feb. 2005.

[19] J. Chung, C.-S. Hwang, K. Kim, and Y. K. Kim, "A random beamforming technique in MIMO systems exploiting multiuser diversity," *IEEE Jour. Select. Areas in Comm.*, vol. 21, p. 848 855, June 2003.

[20] M. Kountouris and D. Gesbert, "Memory-based opportunistic multi-user beamforming," *Proc. IEEE Int. Symp. Info. Th.*, pp. 1426–1430, Sep. 2005.

[21] K. Zyczkowski and M. Kus, "Random unitary matrices," *J. Phys.*, vol. A 27, pp. 4235–4245, 1994.

[22] X. Liu, E. K. Chong, and N. B. Shroff, "Opportunistic transmission scheduling with resource-sharing constraints in wireless networks," *IEEE Jour. Select. Areas in Comm.*, vol. 19, pp. 2053–2064, Oct 2001.

[23] R. W. Heath, M. Airy, and A. J. Paulraj, "Multiuser diversity for MIMO wireless systems with linear receivers," *Proc. of Asilomar Conf. on Sign., Syst. and Computers*, pp. 1194 –1199, Nov 2001.

[24] M. Airy, S. Shakkottai, and R. W. Heath, "Scheduling for the multiple antenna broadcast channel," *submitted to IEEE Trans. on Veh. Technol.*, Oct 2004.

[25] H. A. David and H. N. Nagaraja, *Order statistics*. New Jersey: Wiley, 2003.

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